NAG Library Routine Document

S15ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S15ACF returns the value of the complement of the cumulative Normal distribution function, $Q(x)$, via the function name.

2 Specification

```
FUNCTION S15ACF (X, IFAIL)
REAL (KIND=nag_wp) S15ACF
INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S15ACF evaluates an approximate value for the complement of the cumulative Normal distribution function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2}\, du.$$

The routine is based on the fact that

$$Q(x) = \tfrac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

and it calls S15ADF to obtain the necessary value of *erfc*, the complementary error function.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from this routine. The argument IFAIL is included for consistency with other routines in this chapter.

7 Accuracy

Because of its close relationship with *erfc* the accuracy of this routine is very similar to that in S15ADF. If ϵ and δ are the relative errors in result and argument, respectively, then in principle they are related by

$$|\epsilon| \simeq \left| \frac{xe^{-x^2/2}}{\sqrt{2\pi}Q(x)}\delta \right|.$$

For x negative or small positive this factor is always less than one and accuracy is mainly limited by ***machine precision***. For large positive x we find $\epsilon \sim x^2\delta$ and hence to a certain extent relative accuracy is unavoidably lost. However the absolute error in the result, E, is given by

$$|E| \simeq \left| \frac{xe^{-x^2/2}}{\sqrt{2\pi}}\delta \right|$$

and since this factor is always less than one absolute accuracy can be guaranteed for all x.

8 Parallelism and Performance

S15ACF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
    Program s15acfe

!     S15ACF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s15acf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S15ACF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)
```

```
data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = 0
        y = s15acf(x,ifail)

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s15acfe
```

10.2 Program Data

```
S15ACF Example Program Data
          -20.0
           -1.0
            0.0
            1.0
            2.0
           20.0
```

10.3 Program Results

```
 S15ACF Example Program Results

      X           Y

   -2.000E+01   1.000E+00
   -1.000E+00   8.413E-01
    0.000E+00   5.000E-01
    1.000E+00   1.587E-01
    2.000E+00   2.275E-02
    2.000E+01   2.754E-89
```