

NAG Library Routine Document

S15ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S15ABF returns the value of the cumulative Normal distribution function, $P(x)$, via the function name.

2 Specification

```
FUNCTION S15ABF (X, IFAIL)
  REAL (KIND=nag_wp) S15ABF
  INTEGER IFAIL
  REAL (KIND=nag_wp) X
```

3 Description

S15ABF evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du.$$

The routine is based on the fact that

$$P(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{-x}{\sqrt{2}}\right)$$

and it calls S15ADF to obtain a value of *erfc* for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1 . If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from this routine. The argument IFAIL is included for consistency with other routines in this chapter.

7 Accuracy

Because of its close relationship with *erfc*, the accuracy of this routine is very similar to that in S15ADF. If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)} \delta \right|$$

so that the relative error in the argument, x , is amplified by a factor, $\frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by *machine precision*.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E , is given by

$$|E| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \delta \right|$$

so absolute accuracy can be guaranteed for all x .

8 Parallelism and Performance

S15ABF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```

Program s15abfe

!      S15ABF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: nag_wp, s15abf
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)         :: x, y
      Integer                    :: ifail, ioerr

```

```

!      .. Executable Statements ..
      Write (nout,*) 'S15ABF Example Program Results'

!      Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '          X              Y'
      Write (nout,*)

data: Do
      Read (nin,*,Iostat=ioerr) x

      If (ioerr<0) Then
        Exit data
      End If

      ifail = 0
      y = s15abf(x,ifail)

      Write (nout,99999) x, y
End Do data

99999 Format (1X,1P,2E12.3)
End Program s15abfe

```

10.2 Program Data

```

S15ABF Example Program Data
      -20.0
      -1.0
       0.0
       1.0
       2.0
      20.0

```

10.3 Program Results

S15ABF Example Program Results

X	Y
-2.000E+01	2.754E-89
-1.000E+00	1.587E-01
0.000E+00	5.000E-01
1.000E+00	8.413E-01
2.000E+00	9.772E-01
2.000E+01	1.000E+00
