NAG Library Routine Document

S13ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S13ADF returns the value of the sine integral

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\,du,$$

via the function name.

2 Specification

```
FUNCTION S13ADF (X, IFAIL)
REAL (KIND=nag_wp) S13ADF
INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S13ADF calculates an approximate value for $\mathrm{Si}(x)$.

For $|x| \leq 16.0$ it is based on the Chebyshev expansion

$$\mathrm{Si}(x) = x{\sum_{r=0}}' a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < |x| < x_{\mathrm{hi}}$, where x_{hi} is an implementation-dependent number,

$$\mathrm{Si}(x) = \mathrm{sign}(x)\left\{\frac{\pi}{2} - \frac{f(x)\cos x}{x} - \frac{g(x)\sin x}{x^2}\right\}$$

where $f(x) = \sum_{r=0}{}' f_r T_r(t)$ and $g(x) = \sum_{r=0}{}' g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $|x| \geq x_{\mathrm{hi}}$, $\mathrm{Si}(x) = \frac{1}{2}\pi \,\mathrm{sign}\, x$ to within ***machine precision***.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then

the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from S13ADF. The argument IFAIL has been included for consistency with other routines in this chapter.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left|\frac{\delta \sin x}{\mathrm{Si}(x)}\right|.$$

The equality may hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation, then since the factor relating δ to ϵ is always less than one, the accuracy will be limited by ***machine precision***.

8 Parallelism and Performance

S13ADF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
    Program s13adfe

!     S13ADF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s13adf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S13ADF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)
```

```
data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s13adf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s13adfe
```

10.2 Program Data

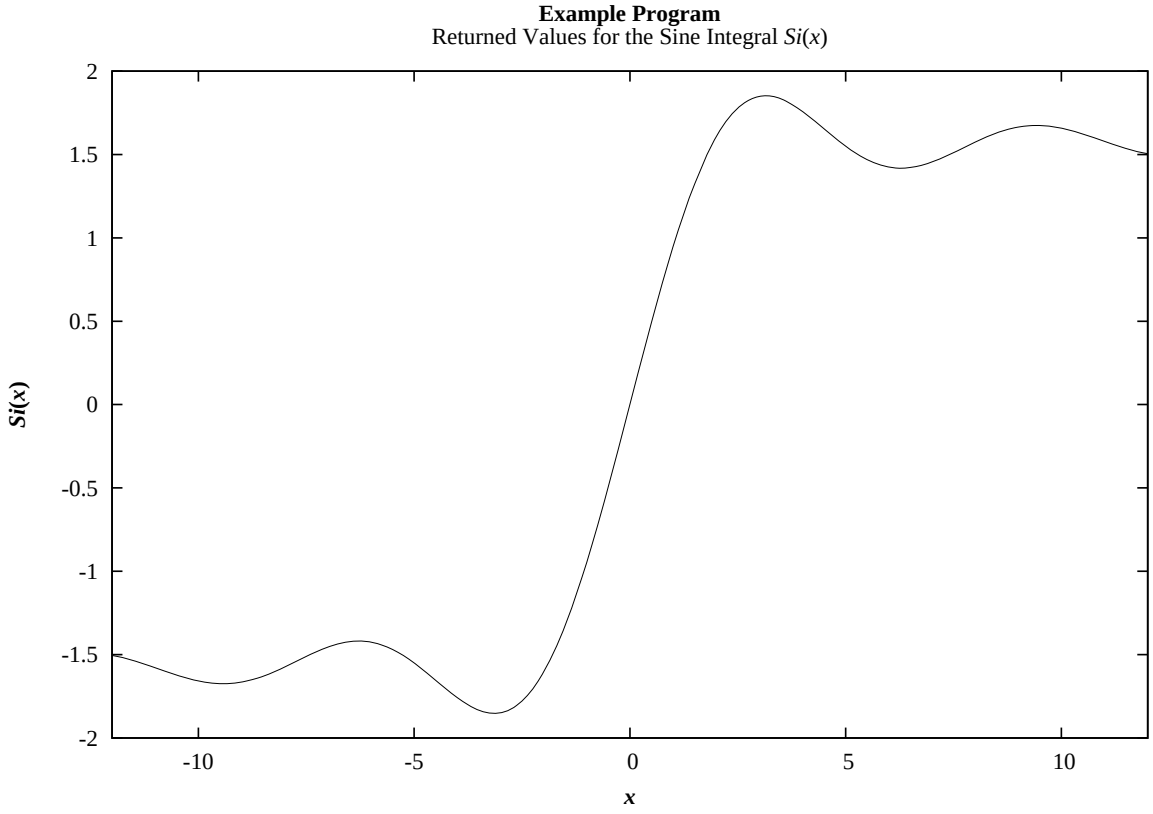
```
S13ADF Example Program Data
             0.0
             0.2
             0.4
             0.6
             0.8
             1.0
```

10.3 Program Results

```
 S13ADF Example Program Results

      X           Y

    0.000E+00   0.000E+00
    2.000E-01   1.996E-01
    4.000E-01   3.965E-01
    6.000E-01   5.881E-01
    8.000E-01   7.721E-01
    1.000E+00   9.461E-01
```


Example Program
Returned Values for the Sine Integral Si(x)
Si(x)
x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-5
0
5
10