NAG Library Routine Document

S11ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S11ACF returns the value of the inverse hyperbolic cosine, $\operatorname{arccosh} x$, via the function name. The result is in the principal positive branch.

2 Specification

```
FUNCTION S11ACF (X, IFAIL)
REAL (KIND=nag_wp) S11ACF
INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S11ACF calculates an approximate value for the inverse hyperbolic cosine, $\operatorname{arccosh} x$. It is based on the relation

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right).$$

This form is used directly for $1 < x < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $x \geq 10^k$, $\sqrt{x^2 - 1}$ is equal to $\sqrt{x}$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arccosh} x = \ln 2 + \ln x.$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

Constraint: $X \geq 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: $\text{IFAIL} = 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument less than 1.0, for which $\operatorname{arccosh} x$ is not defined. The result returned is zero.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x} \times \delta \right|.$$

That is the relative error in the argument is amplified by a factor at least $\frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x}$ in the result.

The equality should apply if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation it is possible that an extra figure may be lost in internal calculation and round-off. The behaviour of the amplification factor is shown in the following graph:

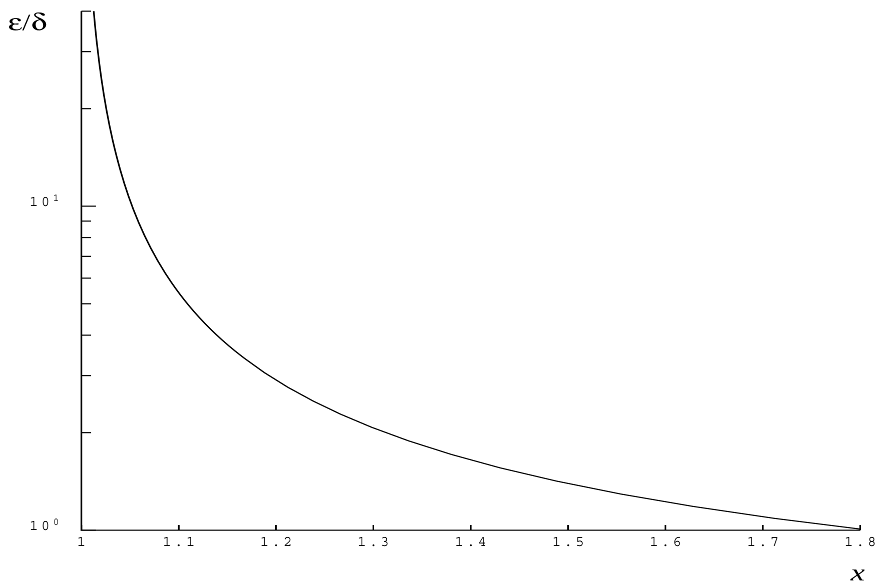

Figure 1

It should be noted that for $x > 2$ the factor is always less than 1.0. For large x we have the absolute error E in the result, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***. More significantly for x close to 1, $x - 1 \sim \delta$, the above analysis becomes inapplicable due to the fact that both function and argument are bounded, $x \geq 1$, $\operatorname{arccosh} x \geq 0$. In this region we have

$$E \sim \sqrt{\delta}.$$

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.

8 Parallelism and Performance

S11ACF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
    Program s11acfe

!     S11ACF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s11acf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
```

```
!     .. Local Scalars ..
      Real (Kind=nag_wp)                :: x, y
      Integer                           :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S11ACF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X             Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s11acf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s11acfe
```

10.2 Program Data

```
S11ACF Example Program Data
             1.00
             2.0
             5.0
            10.0
```

10.3 Program Results

```
 S11ACF Example Program Results

      X             Y

    1.000E+00   0.000E+00
    2.000E+00   1.317E+00
    5.000E+00   2.292E+00
    1.000E+01   2.993E+00
```