

NAG Library Routine Document

S09ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S09ABF returns the value of the inverse circular cosine, $\arccos x$, via the function name; the result is in the principal range $(0, \pi)$.

2 Specification

```
FUNCTION S09ABF (X, IFAIL)
REAL (KIND=nag_wp) S09ABF
INTEGER IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S09ABF calculates an approximate value for the inverse circular cosine, $\arccos x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(t) = x \sum_{r=0} a_r T_r(t)$$

where $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$, and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arccos x = \frac{\pi}{2} - \arcsin x$.

For $-1 \leq x < \frac{-1}{\sqrt{2}}$, $\arccos x = \pi - \arcsin \sqrt{1 - x^2}$.

For $\frac{1}{\sqrt{2}} < x \leq 1$, $\arccos x = \arcsin \sqrt{1 - x^2}$.

For $|x| > 1$, $\arccos x$ is undefined and the routine fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

- 1: X – REAL (KIND=nag_wp) *Input*
On entry: the argument x of the function.
Constraint: $|X| \leq 1.0$.
- 2: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then

the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

S09ABF has been called with $|X| > 1.0$, for which arccos is undefined. A zero result is returned.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arccos x \sqrt{1-x^2}} \times \delta \right|.$$

The equality should hold if δ is greater than the **machine precision** (δ is due to data errors etc.), but if δ is due simply to round-off in the machine it is possible that rounding etc. in internal calculations may lose one extra figure.

The behaviour of the amplification factor $\frac{x}{\arccos x \sqrt{1-x^2}}$ is shown in the graph below.

In the region of $x = 0$ this factor tends to zero and the accuracy will be limited by the **machine precision**. For $|x|$ close to one, $1 - |x| \sim \delta$, the above analysis is not applicable owing to the fact that both the argument and the result are bounded $|x| \leq 1$, $0 \leq \arccos x \leq \pi$.

In the region of $x \sim -1$ we have $\epsilon \sim \sqrt{\delta}$, that is the result will have approximately half as many correct significant figures as the argument.

In the region $x \sim +1$, we have that the absolute error in the result, E , is given by $E \sim \sqrt{\delta}$, that is the result will have approximately half as many decimal places correct as there are correct figures in the argument.

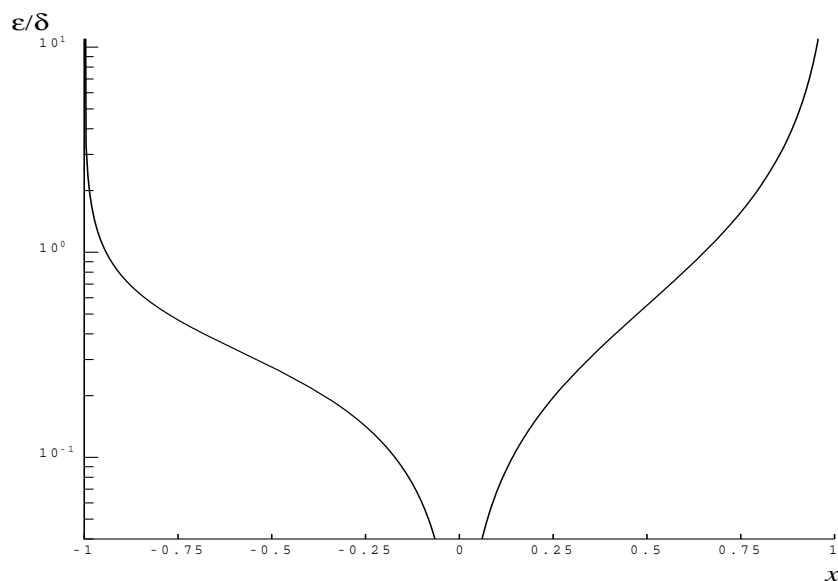


Figure 1

8 Parallelism and Performance

S09ABF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```

Program s09abfe

!      S09ABF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: nag_wp, s09abf
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)          :: x, y
      Integer                      :: ifail, ioerr
!      .. Executable Statements ..
      Write (nout,*) 'S09ABF Example Program Results'

!      Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '      X          Y'
      Write (nout,*)

data: Do

```

```
      Read (nin,*,Iostat=ioerr) x

      If (ioerr<0) Then
        Exit data
      End If

      ifail = -1
      y = s09abf(x,ifail)

      If (ifail<0) Then
        Exit data
      End If

      Write (nout,99999) x, y
    End Do data

99999 Format (1X,1P,2E12.3)
End Program s09abfe
```

10.2 Program Data

S09ABF Example Program Data

-0.5
0.1
0.9

10.3 Program Results

S09ABF Example Program Results

X	Y
-5.000E-01	2.094E+00
1.000E-01	1.471E+00
9.000E-01	4.510E-01
