

# NAG Library Routine Document

## G13FAF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

G13FAF estimates the parameters of either a standard univariate regression GARCH process, or a univariate regression-type I AGARCH( $p, q$ ) process (see Engle and Ng (1993)).

### 2 Specification

```
SUBROUTINE G13FAF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, ISYM, NPAR,      &
                  THETA, SE, SC, COVR, LDCOVR, HP, ET, HT, LGF, COPTS,      &
                  MAXIT, TOL, WORK, LWORK, IFAIL)

INTEGER          LDX, NUM, IP, IQ, NREG, MN, ISYM, NPAR, LDCOVR,          &
                  MAXIT, LWORK, IFAIL
REAL (KIND=nag_wp) YT(NUM), X(LDX,*), THETA(NPAR), SE(NPAR), SC(NPAR),    &
                  COVR(LDCOVR,NPAR), HP, ET(NUM), HT(NUM), LGF, TOL,      &
                  WORK(LWORK)
LOGICAL          COPTS(2)
CHARACTER(1)     DIST
```

### 3 Description

A univariate regression-type I AGARCH( $p, q$ ) process, with  $q$  coefficients  $\alpha_i$ , for  $i = 1, 2, \dots, q$ ,  $p$  coefficients  $\beta_i$ , for  $i = 1, 2, \dots, p$ , and  $k$  linear regression coefficients  $b_i$ , for  $i = 1, 2, \dots, k$ , can be represented by:

$$y_t = b_o + x_t^T b + \epsilon_t \quad (1)$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i (\epsilon_{t-i} + \gamma)^2 + \sum_{i=1}^p \beta_i h_{t-i}, \quad t = 1, 2, \dots, T \quad (2)$$

where  $\epsilon_t | \psi_{t-1} = N(0, h_t)$  or  $\epsilon_t | \psi_{t-1} = S_t(df, h_t)$ . Here  $S_t$  is a standardized Student's  $t$ -distribution with  $df$  degrees of freedom and variance  $h_t$ ,  $T$  is the number of terms in the sequence,  $y_t$  denotes the endogenous variables,  $x_t$  the exogenous variables,  $b_o$  the regression mean,  $b$  the regression coefficients,  $\epsilon_t$  the residuals,  $h_t$  the conditional variance,  $df$  the number of degrees of freedom of the Student's  $t$ -distribution, and  $\psi_t$  the set of all information up to time  $t$ .

G13FAF provides an estimate for  $\hat{\theta}$ , the parameter vector  $\theta = (b_o, b^T, \omega^T)$  where  $b^T = (b_1, \dots, b_k)$ ,  $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma)$  when DIST = 'N' and  $\omega^T = (\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1, \dots, \beta_p, \gamma, df)$  when DIST = 'T'.

ISYM, MN and NREG can be used to simplify the GARCH( $p, q$ ) expression in (1) as follows:

#### No Regression and No Mean

$$y_t = \epsilon_t,$$

$$\text{ISYM} = 0,$$

$$\text{MN} = 0,$$

$$\text{NREG} = 0 \text{ and}$$

$\theta$  is a  $(p + q + 1)$  vector when DIST = 'N' and a  $(p + q + 2)$  vector when DIST = 'T'.

**No Regression**

$$y_t = b_o + \epsilon_t,$$

$$\text{ISYM} = 0,$$

$$\text{MN} = 1,$$

$$\text{NREG} = 0 \text{ and}$$

$\theta$  is a  $(p + q + 2)$  vector when  $\text{DIST} = \text{'N'}$  and a  $(p + q + 3)$  vector when  $\text{DIST} = \text{'T'}$ .

**Note:** if the  $y_t = \mu + \epsilon_t$ , where  $\mu$  is known (not to be estimated by G13FAF) then (1) can be written as  $y_t^\mu = \epsilon_t$ , where  $y_t^\mu = y_t - \mu$ . This corresponds to the case **No Regression and No Mean**, with  $y_t$  replaced by  $y_t - \mu$ .

**No Mean**

$$y_t = x_t^T b + \epsilon_t,$$

$$\text{ISYM} = 0,$$

$$\text{MN} = 0,$$

$$\text{NREG} = k \text{ and}$$

$\theta$  is a  $(p + q + k + 1)$  vector when  $\text{DIST} = \text{'N'}$  and a  $(p + q + k + 2)$  vector when  $\text{DIST} = \text{'T'}$ .

**4 References**

Bollerslev T (1986) Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Engle R and Ng V (1993) Measuring and testing the impact of news on volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

**5 Arguments**

- 1: DIST – CHARACTER(1) *Input*  
*On entry:* the type of distribution to use for  $e_t$ .  
DIST = 'N'  
A Normal distribution is used.  
DIST = 'T'  
A Student's  $t$ -distribution is used.  
*Constraint:* DIST = 'N' or 'T'.
- 2: YT(NUM) – REAL (KIND=nag\_wp) array *Input*  
*On entry:* the sequence of observations,  $y_t$ , for  $t = 1, 2, \dots, T$ .
- 3: X(LDX,\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the second dimension of the array X must be at least NREG.  
*On entry:* row  $t$  of X must contain the time dependent exogenous vector  $x_t$ , where  $x_t^T = (x_t^1, \dots, x_t^k)$ , for  $t = 1, 2, \dots, T$ .

- 4: LDX – INTEGER *Input*  
*On entry:* the first dimension of the array X as declared in the (sub)program from which G13FAF is called.  
*Constraint:*  $LDX \geq \text{NUM}$ .
- 5: NUM – INTEGER *Input*  
*On entry:*  $T$ , the number of terms in the sequence.  
*Constraints:*  

$$\text{NUM} \geq \max(\text{IP}, \text{IQ});$$

$$\text{NUM} \geq \text{NREG} + \text{MN}.$$
- 6: IP – INTEGER *Input*  
*On entry:* the number of coefficients,  $\beta_i$ , for  $i = 1, 2, \dots, p$ .  
*Constraint:*  $\text{IP} \geq 0$  (see also NPAR).
- 7: IQ – INTEGER *Input*  
*On entry:* the number of coefficients,  $\alpha_i$ , for  $i = 1, 2, \dots, q$ .  
*Constraint:*  $\text{IQ} \geq 1$  (see also NPAR).
- 8: NREG – INTEGER *Input*  
*On entry:*  $k$ , the number of regression coefficients.  
*Constraint:*  $\text{NREG} \geq 0$  (see also NPAR).
- 9: MN – INTEGER *Input*  
*On entry:* if  $\text{MN} = 1$ , the mean term  $b_0$  will be included in the model.  
*Constraint:*  $\text{MN} = 0$  or  $1$ .
- 10: ISYM – INTEGER *Input*  
*On entry:* if  $\text{ISYM} = 1$ , the asymmetry term  $\gamma$  will be included in the model.  
*Constraint:*  $\text{ISYM} = 0$  or  $1$ .
- 11: NPAR – INTEGER *Input*  
*On entry:* the number of parameters to be included in the model.  

$$\text{NPAR} = 1 + \text{IQ} + \text{IP} + \text{ISYM} + \text{MN} + \text{NREG} \quad \text{when} \quad \text{DIST} = \text{'N'}, \quad \text{and}$$

$$\text{NPAR} = 2 + \text{IQ} + \text{IP} + \text{ISYM} + \text{MN} + \text{NREG} \quad \text{when} \quad \text{DIST} = \text{'T'}.$$
*Constraint:*  $\text{NPAR} < 20$ .
- 12: THETA(NPAR) – REAL (KIND=nag\_wp) array *Input/Output*  
*On entry:* the initial parameter estimates for the vector  $\theta$ .  
The first element must contain the coefficient  $\alpha_0$  and the next IQ elements must contain the coefficients  $\alpha_i$ , for  $i = 1, 2, \dots, q$ .  
The next IP elements must contain the coefficients  $\beta_j$ , for  $j = 1, 2, \dots, p$ .  
If  $\text{ISYM} = 1$ , the next element must contain the asymmetry parameter  $\gamma$ .  
If  $\text{DIST} = \text{'T'}$ , the next element must contain  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution.  
If  $\text{MN} = 1$ , the next element must contain the mean term  $b_0$ .

If COPTS(2) = .FALSE., the remaining NREG elements are taken as initial estimates of the linear regression coefficients  $b_i$ , for  $i = 1, 2, \dots, k$ .

*On exit:* the estimated values  $\hat{\theta}$  for the vector  $\theta$ .

The first element contains the coefficient  $\alpha_o$ , the next IQ elements contain the coefficients  $\alpha_i$ , for  $i = 1, 2, \dots, q$ .

The next IP elements are the coefficients  $\beta_j$ , for  $j = 1, 2, \dots, p$ .

If ISYM = 1, the next element contains the estimate for the asymmetry parameter  $\gamma$ .

If DIST = 'T', the next element contains an estimate for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution.

If MN = 1, the next element contains an estimate for the mean term  $b_o$ .

The final NREG elements are the estimated linear regression coefficients  $b_i$ , for  $i = 1, 2, \dots, k$ .

- 13: SE(NPAR) – REAL (KIND=nag\_wp) array *Output*

*On exit:* the standard errors for  $\hat{\theta}$ .

The first element contains the standard error for  $\alpha_o$ . The next IQ elements contain the standard errors for  $\alpha_i$ , for  $i = 1, 2, \dots, q$ . The next IP elements are the standard errors for  $\beta_j$ , for  $j = 1, 2, \dots, p$ .

If ISYM = 1, the next element contains the standard error for  $\gamma$ .

If DIST = 'T', the next element contains the standard error for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution.

If MN = 1, the next element contains the standard error for  $b_o$ .

The final NREG elements are the standard errors for  $b_j$ , for  $j = 1, 2, \dots, k$ .

- 14: SC(NPAR) – REAL (KIND=nag\_wp) array *Output*

*On exit:* the scores for  $\hat{\theta}$ .

The first element contains the score for  $\alpha_o$ .

The next IQ elements contain the score for  $\alpha_i$ , for  $i = 1, 2, \dots, q$ .

The next IP elements are the scores for  $\beta_j$ , for  $j = 1, 2, \dots, p$ .

If ISYM = 1, the next element contains the score for  $\gamma$ .

If DIST = 'T', the next element contains the score for  $df$ , the number of degrees of freedom of the Student's  $t$ -distribution.

If MN = 1, the next element contains the score for  $b_o$ .

The final NREG elements are the scores for  $b_j$ , for  $j = 1, 2, \dots, k$ .

- 15: COVR(LDCOVR,NPAR) – REAL (KIND=nag\_wp) array *Output*

*On exit:* the covariance matrix of the parameter estimates  $\hat{\theta}$ , that is the inverse of the Fisher Information Matrix.

- 16: LDCOVR – INTEGER *Input*

*On entry:* the first dimension of the array COVR as declared in the (sub)program from which G13FAF is called.

*Constraint:* LDCOVR  $\geq$  NPAR.

- 17: HP – REAL (KIND=nag\_wp) Input/Output  
*On entry:* if COPTS(2) = .FALSE., HP is the value to be used for the pre-observed conditional variance; otherwise HP is not referenced.  
*On exit:* if COPTS(2) = .TRUE., HP is the estimated value of the pre-observed conditional variance.
- 18: ET(NUM) – REAL (KIND=nag\_wp) array Output  
*On exit:* the estimated residuals,  $\epsilon_t$ , for  $t = 1, 2, \dots, T$ .
- 19: HT(NUM) – REAL (KIND=nag\_wp) array Output  
*On exit:* the estimated conditional variances,  $h_t$ , for  $t = 1, 2, \dots, T$ .
- 20: LGF – REAL (KIND=nag\_wp) Output  
*On exit:* the value of the log-likelihood function at  $\hat{\theta}$ .
- 21: COPTS(2) – LOGICAL array Input  
*On entry:* the options to be used by G13FAF.  
COPTS(1) = .TRUE.  
Stationary conditions are enforced, otherwise they are not.  
COPTS(2) = .TRUE.  
The routine provides initial parameter estimates of the regression terms, otherwise these are to be provided by you.
- 22: MAXIT – INTEGER Input  
*On entry:* the maximum number of iterations to be used by the optimization routine when estimating the GARCH( $p, q$ ) parameters. If MAXIT is set to 0, the standard errors, score vector and variance-covariance are calculated for the input value of  $\theta$  in THETA when DIST = 'N'; however the value of  $\theta$  is not updated.  
*Constraint:* MAXIT  $\geq 0$ .
- 23: TOL – REAL (KIND=nag\_wp) Input  
*On entry:* the tolerance to be used by the optimization routine when estimating the GARCH( $p, q$ ) parameters.
- 24: WORK(LWORK) – REAL (KIND=nag\_wp) array Workspace  
25: LWORK – INTEGER Input  
*On entry:* the dimension of the array WORK as declared in the (sub)program from which G13FAF is called.  
*Constraint:* LWORK  $\geq (\text{NREG} + 3) \times \text{NUM} + \text{NPAR} + 403$ .
- 26: IFAIL – INTEGER Input/Output  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.  
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output arguments may be useful even if IFAIL  $\neq 0$  on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry  $IFAIL = 0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by  $X04AAF$ ).

**Note:** G13FAF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry,  $NREG < 0$ ,  
 or  $MN > 1$ ,  
 or  $MN < 0$ ,  
 or  $ISYM > 1$ ,  
 or  $ISYM < 0$ ,  
 or  $IQ < 1$ ,  
 or  $IP < 0$ ,  
 or  $NPAR \geq 20$ ,  
 or  $NPAR$  has an invalid value,  
 or  $LDCOVR < NPAR$ ,  
 or  $LDX < NUM$ ,  
 or  $DIST \neq 'N'$ ,  
 or  $DIST \neq 'T'$ ,  
 or  $MAXIT < 0$ ,  
 or  $NUM < \max(IP, IQ)$ ,  
 or  $NUM < NREG + MN$ .

$IFAIL = 2$

On entry,  $LWORK < (NREG + 3) \times NUM + NPAR + 403$ .

$IFAIL = 3$

The matrix  $X$  is not full rank.

$IFAIL = 4$

The information matrix is not positive definite.

$IFAIL = 5$

The maximum number of iterations has been reached.

$IFAIL = 6$

The log-likelihood cannot be optimized any further.

$IFAIL = 7$

No feasible model parameters could be found.

$IFAIL = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

$IFAIL = -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

Not applicable.

## 8 Parallelism and Performance

G13FAF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G13FAF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

None.

## 10 Example

This example fits a GARCH(1,1) model with Student's  $t$ -distributed residuals to some simulated data.

The process parameter estimates,  $\hat{\theta}$ , are obtained using G13FAF, and a four step ahead volatility estimate is computed using G13FBF.

The data was simulated using G05PDF.

### 10.1 Program Text

Program g13faf

```
!      G13FAF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: g13faf, g13fbf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)          :: gamma, hp, lgf, tol
      Integer                     :: i, ifail, ip, iq, isym, l, ldcovr, &
                                   ldx, lwork, maxit, mn, npar, nreg, &
                                   nt, num, pgamma
      Logical                     :: tdist
      Character (1)               :: dist
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: covr(:, :), et(:, :), fht(:, :), ht(:, :), &
                                   sc(:, :), se(:, :), theta(:, :), work(:, :), &
                                   x(:, :), yt(:, :),
      Logical                     :: copts(2)
!      .. Executable Statements ..
      Write (nout,*) 'G13FAF Example Program Results'
      Write (nout,*)
```

```

!      Skip heading in data file
      Read (nin,*)

!      Read in the problem size
      Read (nin,*) num, mn, nreg

      ldx = num
      Allocate (yt(num),x(ldx,nreg))

!      Read in the series
      Read (nin,*) yt(1:num)

!      Read in the exogenous variables
      If (nreg>0) Then
        Read (nin,*,Iostat=ifail)(x(i,1:nreg),i=1,num)
      End If

!      Read in details of the model to fit
      Read (nin,*) dist, ip, iq, isym

!      Read in control parameters
      Read (nin,*) copts(1:2), maxit, tol

!      Calculate NPAR
      npar = 1 + iq + ip + isym + mn + nreg
      If (dist=='T' .Or. dist=='t') Then
        npar = npar + 1
        tdist = .True.
      Else
        tdist = .False.
      End If

      ldcovr = npar
      lwork = (nreg+3)*num + npar + 403
      Allocate (theta(npar),se(npar),sc(npar),covr(ldcovr,npar),et(num),      &
        ht(num),work(lwork))

!      Read in initial values
!      alpha_0
      Read (nin,*) theta(1)
      l = 2
!      alpha_i
      If (iq>0) Then
        Read (nin,*) theta(1:(l+iq-1))
        l = l + iq
      End If
!      beta_i
      If (ip>0) Then
        Read (nin,*) theta(1:(l+ip-1))
        l = l + ip
      End If
!      gamma
      If (isym==1) Then
        Read (nin,*) theta(1)
        pgamma = 1
        l = l + 1
      End If
!      degrees of freedom
      If (tdist) Then
        Read (nin,*) theta(1)
        l = l + 1
      End If
!      mean
      If (mn==1) Then
        Read (nin,*) theta(1)
        l = l + 1
      End If
!      Regression parameters and pre-observed conditional variance
      If (.Not. copts(2)) Then
        Read (nin,*) theta(1:(l+nreg-1))

```





```

      Write (nout,99997) 'Volatility forecast = ', fht(nt)
      Write (nout,*)

100   Continue

99999 Format (1X,A,I0,1X,2F16.2)
99998 Format (1X,A,1X,2F16.2)
99997 Format (1X,A,F12.2)
      End Program g13fafa

```

## 10.2 Program Data

```

G13FAF Example Program Data
100 1 2                                :: NUM,MN,NREG
  9.04  9.49  9.12  9.23  9.35
  9.09  9.75  9.23  8.76  9.17
  9.20  9.64  8.74  9.23  9.42
  9.70  9.55 10.00  9.18  9.77
  9.80  9.56  9.28  9.68  9.51
  9.51  8.97  9.30  9.52  9.41
  9.53  9.75  9.72  9.38  9.28
  9.42  9.74  9.75  9.60  9.90
  9.06  9.92  9.21  9.57  9.42
  8.65  8.85  9.61 10.77 10.19
10.47 10.10 10.21  9.96  9.66
  9.79 10.30  9.68 10.08 10.38
  9.69  9.02  9.89 10.46 10.47
  9.99  9.76  9.78  9.62 10.43
10.42  9.95  9.95  9.70 10.24
  9.78  9.98  8.73 10.23  9.10
10.27  9.85 10.44 10.30 10.08
10.20 10.14  9.89  9.90 11.33
  9.71  9.40  9.97 10.92  9.76
10.16 10.43  9.60 10.29 10.03      :: End of Y
  0.12  2.40          0.12  2.40
  0.13  2.40          0.14  2.40
  0.14  2.40          0.15  2.40
  0.16  2.40          0.16  2.40
  0.17  2.40          0.18  2.41
  0.19  2.41          0.19  2.41
  0.20  2.41          0.21  2.41
  0.21  2.41          0.22  2.41
  0.23  2.41          0.23  2.41
  0.24  2.41          0.25  2.42
  0.25  2.42          0.26  2.42
  0.26  2.42          0.27  2.42
  0.28  2.42          0.28  2.42
  0.29  2.42          0.30  2.42
  0.30  2.42          0.31  2.43
  0.32  2.43          0.32  2.43
  0.33  2.43          0.33  2.43
  0.34  2.43          0.35  2.43
  0.35  2.43          0.36  2.43
  0.37  2.43          0.37  2.44
  0.38  2.44          0.38  2.44
  0.39  2.44          0.39  2.44
  0.40  2.44          0.41  2.44
  0.41  2.44          0.42  2.44
  0.42  2.44          0.43  2.45
  0.43  2.45          0.44  2.45
  0.45  2.45          0.45  2.45
  0.46  2.45          0.46  2.45
  0.47  2.45          0.47  2.45
  0.48  2.45          0.48  2.46
  0.49  2.46          0.49  2.46
  0.50  2.46          0.50  2.46
  0.51  2.46          0.51  2.46
  0.52  2.46          0.52  2.46
  0.53  2.46          0.53  2.47
  0.54  2.47          0.54  2.47

```

```

0.54  2.47      0.55  2.47
0.55  2.47      0.56  2.47
0.56  2.47      0.57  2.47
0.57  2.47      0.57  2.48
0.58  2.48      0.58  2.48
0.59  2.48      0.59  2.48
0.59  2.48      0.60  2.48
0.60  2.48      0.61  2.48
0.61  2.48      0.61  2.49
0.62  2.49      0.62  2.49
0.62  2.49      0.63  2.49
0.63  2.49      0.63  2.49
0.64  2.49      0.64  2.49
0.64  2.49      0.64  2.50
'T' 1 1 1
T T 200 0.00001
0.05
0.10
0.15
-0.10
2.60
1.50
4
:: End of X
:: DIST,IP,IQ,ISYM
:: COPTS,MAXIT,TOL
:: ALPHA_0
:: ALPHA_I
:: BETA_I
:: GAMMA
:: DF
:: MEAN
:: NT

```

### 10.3 Program Results

G13FAF Example Program Results

	Parameter estimates	Standard errors
Alpha0	0.00	0.06
Alphal	0.11	0.13
Betal	0.66	0.23
Gamma	-0.62	0.62
DF	6.25	4.70
B0	3.85	24.11
B1	1.48	1.82
B2	2.15	10.16

Volatility forecast = 0.09

---