

NAG Library Routine Document

G13DPF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

G13DPF calculates the sample partial autoregression matrices of a multivariate time series. A set of likelihood ratio statistics and their significance levels are also returned. These quantities are useful for determining whether the series follows an autoregressive model and, if so, of what order.

2 Specification

```
SUBROUTINE G13DPF (K, N, Z, KMAX, M, MAXLAG, PARLAG, SE, QQ, X, PVALUE,      &
                  LOGLHD, WORK, LWORK, IWORK, IFAIL)
INTEGER           K, N, KMAX, M, MAXLAG, LWORK, IWORK(K*M), IFAIL
REAL (KIND=nag_wp) Z(KMAX,N), PARLAG(KMAX,KMAX,M), SE(KMAX,KMAX,M),      &
                  QQ(KMAX,KMAX,M), X(M), PVALUE(M), LOGLHD(M),            &
                  WORK(LWORK)
```

3 Description

Let $W_t = (w_{1t}, w_{2t}, \dots, w_{kt})^T$, for $t = 1, 2, \dots, n$, denote a vector of k time series. The partial autoregression matrix at lag l , P_l , is defined to be the last matrix coefficient when a vector autoregressive model of order l is fitted to the series. P_l has the property that if W_t follows a vector autoregressive model of order p then $P_l = 0$ for $l > p$.

Sample estimates of the partial autoregression matrices may be obtained by fitting autoregressive models of successively higher orders by multivariate least squares; see Tiao and Box (1981) and Wei (1990). These models are fitted using a QR algorithm based on the routines G02DCF and G02DFF. They are calculated up to lag m , which is usually taken to be at most $n/4$.

The routine also returns the asymptotic standard errors of the elements of \hat{P}_l and an estimate of the residual variance-covariance matrix $\hat{\Sigma}_l$, for $l = 1, 2, \dots, m$. If S_l denotes the residual sum of squares and cross-products matrix after fitting an $AR(l)$ model to the series then under the null hypothesis $H_0 : P_l = 0$ the test statistic

$$X_l = -\left((n - m - 1) - \frac{1}{2} - lk\right) \log \left(\frac{|S_l|}{|S_{l-1}|} \right)$$

is asymptotically distributed as χ^2 with k^2 degrees of freedom. X_l provides a useful diagnostic aid in determining the order of an autoregressive model. (Note that $\hat{\Sigma}_l = S_l/(n - l)$.) The routine also returns an estimate of the maximum of the log-likelihood function for each AR model that has been fitted.

4 References

Tiao G C and Box G E P (1981) Modelling multiple time series with applications *J. Am. Stat. Assoc.* **76** 802–816

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Arguments

- 1: K – INTEGER *Input*
On entry: k , the number of time series.
Constraint: $K \geq 1$.

- 2: N – INTEGER *Input*
On entry: n , the number of observations in the time series.
Constraint: $N \geq 4$.

- 3: Z(KMAX,N) – REAL (KIND=nag_wp) array *Input*
On entry: $Z(i, t)$ must contain the observation w_{it} , for $i = 1, 2, \dots, k$ and $t = 1, 2, \dots, n$.

- 4: KMAX – INTEGER *Input*
On entry: the first dimension of the arrays Z, PARLAG, SE and QQ and the second dimension of the arrays PARLAG, SE and QQ as declared in the (sub)program from which G13DPF is called.
Constraint: $KMAX \geq K$.

- 5: M – INTEGER *Input*
On entry: m , the number of partial autoregression matrices to be computed. If in doubt set $M = 10$.
Constraint: $M \geq 1$ and $N - M - (K \times M + 1) \geq K$.

- 6: MAXLAG – INTEGER *Output*
On exit: the maximum lag up to which partial autoregression matrices (along with their likelihood ratio statistics and their significance levels) have been successfully computed. On a successful exit MAXLAG will equal M. If IFAIL = 2 on exit then MAXLAG will be less than M.

- 7: PARLAG(KMAX,KMAX,M) – REAL (KIND=nag_wp) array *Output*
On exit: $PARLAG(i, j, l)$ contains an estimate of the (i, j) th element of the partial autoregression matrix at lag l , $\hat{P}_l(ij)$, for $l = 1, 2, \dots, MAXLAG$, $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, k$.

- 8: SE(KMAX,KMAX,M) – REAL (KIND=nag_wp) array *Output*
On exit: $SE(i, j, l)$ contains an estimate of the standard error of the corresponding element in the array PARLAG.

- 9: QQ(KMAX,KMAX,M) – REAL (KIND=nag_wp) array *Output*
On exit: $QQ(i, j, l)$ contains an estimate of the (i, j) th element of the corresponding variance-covariance matrix $\hat{\Sigma}_l$, for $l = 1, 2, \dots, MAXLAG$, $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, k$.

- 10: X(M) – REAL (KIND=nag_wp) array *Output*
On exit: $X(l)$ contains X_l , the likelihood ratio statistic at lag l , for $l = 1, 2, \dots, MAXLAG$.

- 11: PVALUE(M) – REAL (KIND=nag_wp) array *Output*
On exit: $PVALUE(l)$ contains the significance level of the statistic in the corresponding element of X.

- 12: LOGLHD(M) – REAL (KIND=nag_wp) array *Output*
On exit: LOGLHD(l) contains an estimate of the maximum of the log-likelihood function when an AR(l) model has been fitted to the series, for $l = 1, 2, \dots, \text{MAXLAG}$.
- 13: WORK(LWORK) – REAL (KIND=nag_wp) array *Workspace*
 14: LWORK – INTEGER *Input*
On entry: the dimension of the array WORK as declared in the (sub)program from which G13DPF is called.
Constraint: $\text{LWORK} \geq (k+1)k + l(4+k) + 2l^2$, where $l = mk + 1$.
- 15: IWORK(K × M) – INTEGER array *Workspace*
- 16: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $K < 1$,
 or $N < 4$,
 or $K_{\text{MAX}} < K$,
 or $M < 1$,
 or $N - M - (K \times M + 1) < K$,
 or LWORK is too small.

IFAIL = 2

The recursive equations used to compute the sample partial autoregression matrices have broken down at lag $\text{MAXLAG} + 1$. This exit could occur if the regression model is overparameterised. For your settings of k and n the value returned by MAXLAG is the largest permissible value of m for which the model is not overparameterised. All output quantities in the arrays PARLAG, SE, QQ, X, PVALUE and LOGLHD up to and including lag MAXLAG will be correct.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computations are believed to be stable.

8 Parallelism and Performance

G13DPF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G13DPF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The time taken is roughly proportional to nmk .

For each order of autoregressive model that has been estimated, G13DPF returns the maximum of the log-likelihood function. An alternative means of choosing the order of a vector AR process is to choose the order for which Akaike's information criterion is smallest. That is, choose the value of l for which $-2 \times \text{LOGLHD}(l) + 2lk^2$ is smallest. You should be warned that this does not always lead to the same choice of l as indicated by the sample partial autoregression matrices and the likelihood ratio statistics.

10 Example

This example computes the sample partial autoregression matrices of two time series of length 48 up to lag 10.

10.1 Program Text

```
! G13DPF Example Program Text
! Mark 26 Release. NAG Copyright 2016.

Module g13dpfe_mod

! G13DPF Example Program Module:
! Parameters and User-defined Routines

! .. Use Statements ..
Use nag_library, Only: nag_wp
! .. Implicit None Statement ..
Implicit None
! .. Accessibility Statements ..
Private
Public :: zprint
! .. Parameters ..
Integer, Parameter, Public :: nin = 5, nout = 6
Contains
Subroutine zprint(k,m,ldpar,maxlag,parlag,se,qq,x,pvalue,nout,ifail)

! .. Scalar Arguments ..
Integer, Intent (In) :: ifail, k, ldpar, m, maxlag, nout
! .. Array Arguments ..
Real (Kind=nag_wp), Intent (In) :: parlag(ldpar,ldpar,m), pvalue(m), &
```

```

                                qq(ldpar,ldpar,m),          &
                                se(ldpar,ldpar,m), x(m)
!
! .. Local Scalars ..
Real (Kind=nag_wp)           :: sum
Integer                      :: i, i2, j, l
!
! .. Local Arrays ..
Character (6)                 :: st(6)
!
! .. Executable Statements ..
Display titles
If (k>1) Then
  Write (nout,99999)
Else If (k==1) Then
  Write (nout,99998)
End If

Do l = 1, maxlag
  Do j = 1, k
    sum = parlag(1,j,l)
    st(j) = '.'
    If (sum>1.96E0_nag_wp*se(1,j,l)) Then
      st(j) = '+'
    End If
    If (sum<-1.96E0_nag_wp*se(1,j,l)) Then
      st(j) = '-'
    End If
  End Do
  If (k==1) Then
    Write (nout,99997) l, (parlag(1,j,l),j=1,k), (st(i2),i2=1,k), &
      qq(1,1,l), x(1), pvalue(1)
    Write (nout,99996)(se(1,j,l),j=1,k)
  Else If (k==2) Then
    Write (nout,99995) l, (parlag(1,j,l),j=1,k), (st(i2),i2=1,k), &
      qq(1,1,l), x(1), pvalue(1)
    Write (nout,99994)(se(1,j,l),j=1,k)
  Else If (k==3) Then
    Write (nout,99993) l, (parlag(1,j,l),j=1,k), (st(i2),i2=1,k), &
      qq(1,1,l), x(1), pvalue(1)
    Write (nout,99992)(se(1,j,l),j=1,k)
  Else If (k==4) Then
    Write (nout,99991) l
    Write (nout,99986)(parlag(1,j,l),j=1,k), (st(i2),i2=1,k), &
      qq(1,1,l), x(1), pvalue(1)
    Write (nout,99990)(se(1,j,l),j=1,k)
  End If

  Do i = 2, k
    Do j = 1, k
      sum = parlag(i,j,l)
      st(j) = '.'
      If (sum>1.96E0_nag_wp*se(i,j,l)) Then
        st(j) = '+'
      End If
      If (sum<-1.96E0_nag_wp*se(i,j,l)) Then
        st(j) = '-'
      End If
    End Do
    If (k==2) Then
      Write (nout,99989)(parlag(i,j,l),j=1,k), (st(i2),i2=1,k), &
        qq(i,i,l)
      Write (nout,99994)(se(i,j,l),j=1,k)
    Else If (k==3) Then
      Write (nout,99988)(parlag(i,j,l),j=1,k), (st(i2),i2=1,k), &
        qq(i,i,l)
      Write (nout,99992)(se(i,j,l),j=1,k)
    Else If (k==4) Then
      Write (nout,99987)(parlag(i,j,l),j=1,k), (st(i2),i2=1,k), &
        qq(i,i,l)
      Write (nout,99990)(se(i,j,l),j=1,k)
    End If
  End Do
End Do

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      End Do
    End Do

    If (ifail==2) Then
      Write (nout,99985) 'Recursive equations broke down at ', maxlag + 1
    End If

    Return

99999  Format (' Partial Autoregression Matrices',4X,'Indicator',2X,      &
        'Residual',3X,'Chi-Square',2X,'Pvalue',/,37X,'Symbols',3X,      &
        'Variances',3X,'Statistic',/, '-----',4X,      &
        '-----',2X,'-----',2X,'-----',1X,'-----')
99998  Format (' Partial Autoregression Function',4X,'Indicator',2X,      &
        'Residual',3X,'Chi-Square',2X,'Pvalue',/,37X,'Symbols',3X,      &
        'Variances',3X,'Statistic',/, '-----',4X,      &
        '-----',2X,'-----',2X,'-----',1X,'-----')
99997  Format (/, ' Lag',I3,1X,':',F7.3,22X,A1,F14.3,3X,F10.3,F9.3)
99996  Format (9X, '(' ,F6.3,') ')
99995  Format (/, ' Lag',I3,1X,':',2F8.3,14X,2A1,F13.3,3X,F10.3,F9.3)
99994  Format (10X, '(' ,F6.3,') (' ,F6.3,') ')
99993  Format (/, ' Lag',I3,1X,':',3F8.3,6X,3A1,F12.3,3X,F10.3,F9.3)
99992  Format (10X, '(' ,F6.3,') (' ,F6.3,') (' ,F6.3,') ')
99991  Format (/, ' Lag',I3)
99990  Format (2X, '(' ,F6.3,') (' ,F6.3,') (' ,F6.3,') (' ,F6.3,') ')
99989  Format (9X,2F8.3,14X,2A1,F13.3)
99988  Format (9X,3F8.3,6X,3A1,F12.3)
99987  Format (1X,4F8.3,5X,4A1,F12.3)
99986  Format (1X,4F8.3,5X,4A1,F12.3,3X,F10.3,F9.3)
99985  Format (1X,A,I0)
      End Subroutine zprint
    End Module g13dpfe_mod
  Program g13dpfe

!      G13DPF Example Main Program

!      .. Use Statements ..
      Use nag_library, Only: g13dpf, nag_wp
      Use g13dpfe_mod, Only: nin, nout, zprint
!      .. Implicit None Statement ..
      Implicit None
!      .. Local Scalars ..
      Integer                                :: i, ifail, k, kmax, l, lwork, m,      &
                                         maxlag, mk, n

!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: loglhd(:), parlag(:,:,:), pvalue(:), &
                                         qq(:,:,:), se(:,:,:), work(:), x(:), &
                                         z(:,:)
      Integer, Allocatable :: iwork(:)

!      .. Executable Statements ..
      Write (nout,*) 'G13DPF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)

!      Read in the problem size
      Read (nin,*) k, n, m

      kmax = k
      mk = m*k
      l = mk + 1
      lwork = (k+1)*k + 1*(4+k)*2*1**2
      Allocate (z(kmax,n),parlag(kmax,kmax,m),se(kmax,kmax,m),x(m),pvalue(m), &
              loglhd(m),work(lwork),qq(kmax,kmax,m),iwork(mk))

!      Read in series
      Do i = 1, k
        Read (nin,*) z(i,1:n)
      End Do

```

```

!      Calculate sample partial autoregression matrices
      ifail = -1
      Call g13dpf(k,n,z,kmax,m,maxlag,parlag,se,qq,x,pvalue,loglhd,work,lwork, &
        iwork,ifail)
      If (ifail/=0) Then
        If (ifail/=2) Then
          Go To 100
        End If
      End If

!      Display results
      Call zprint(k,m,kmax,maxlag,parlag,se,qq,x,pvalue,nout,ifail)

100    Continue

      End Program g13dpfe

```

10.2 Program Data

G13DPF Example Program Data

```

2 48 10                                :: K,N,M
-1.490 -1.620  5.200  6.230  6.210  5.860  4.090  3.180
 2.620  1.490  1.170  0.850 -0.350  0.240  2.440  2.580
 2.040  0.400  2.260  3.340  5.090  5.000  4.780  4.110
 3.450  1.650  1.290  4.090  6.320  7.500  3.890  1.580
 5.210  5.250  4.930  7.380  5.870  5.810  9.680  9.070
 7.290  7.840  7.550  7.320  7.970  7.760  7.000  8.350
 7.340  6.350  6.960  8.540  6.620  4.970  4.550  4.810
 4.750  4.760 10.880 10.010 11.620 10.360  6.400  6.240
 7.930  4.040  3.730  5.600  5.350  6.810  8.270  7.680
 6.650  6.080 10.250  9.140 17.750 13.300  9.630  6.800
 4.080  5.060  4.940  6.650  7.940 10.760 11.890  5.850
 9.010  7.500 10.020 10.380  8.150  8.370 10.730 12.140 :: End of Z

```

10.3 Program Results

G13DPF Example Program Results

Partial Autoregression Matrices	Indicator Symbols	Residual Variances	Chi-Square Statistic	Pvalue
-----	-----	-----	-----	-----
Lag 1 : 0.757 0.062 (0.092)(0.092) 0.061 0.570 (0.129)(0.130)	+. .+	2.731 5.440	49.884	0.000
Lag 2 : -0.161 -0.135 (0.145)(0.109) -0.093 -0.065 (0.213)(0.160)	2.530 5.486	3.347	0.502
Lag 3 : 0.237 0.044 (0.128)(0.095) 0.047 -0.248 (0.222)(0.165)	1.755 5.291	13.962	0.007
Lag 4 : -0.098 0.152 (0.134)(0.099) 0.402 -0.194 (0.228)(0.168)	1.661 4.786	7.071	0.132
Lag 5 : 0.257 -0.026 (0.141)(0.106) 0.400 -0.021 (0.242)(0.183)	1.504 4.447	5.184	0.269
Lag 6 : -0.075 0.112 (0.156)(0.111) 0.196 -0.106	1.480 4.425	2.083	0.721

		(0.269)(0.192)				
Lag 7 :	-0.054	0.097	..	1.478	5.074	0.280
	(0.166)	(0.121)				
	0.574	-0.080	+.	3.838		
	(0.267)	(0.195)				
Lag 8 :	0.147	0.041	..	1.415	10.991	0.027
	(0.188)	(0.128)				
	0.916	-0.242	+.	2.415		
	(0.246)	(0.167)				
Lag 9 :	-0.039	0.099	..	1.322	3.936	0.415
	(0.251)	(0.140)				
	-0.500	0.173	..	2.196		
	(0.324)	(0.181)				
Lag 10 :	0.189	0.131	..	1.206	3.175	0.529
	(0.275)	(0.157)				
	-0.183	-0.040	..	2.201		
	(0.371)	(0.212)				
