NAG Library Routine Document

G13CDF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13CDF calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

2 Specification

```
SUBROUTINE G13CDF (NXY, MTXY, PXY, MW, ISH, PW, L, KC, XG, YG, NG,        &
                   IFAIL)

INTEGER            NXY, MTXY, MW, ISH, L, KC, NG, IFAIL
REAL (KIND=nag_wp) PXY, PW, XG(KC), YG(KC)
```

3 Description

The supplied time series may be mean and trend corrected and tapered as in the description of G13CBF before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^*(\omega) = \frac{1}{2\pi n}\left\{\sum_{t=1}^{n} y_t \exp(i\omega t)\right\} \times \left\{\sum_{t=1}^{n} x_t \exp(-i\omega t)\right\}$$

for frequency values $\omega_j = \frac{2\pi j}{K}$, $0 \leq \omega_j \leq \pi$.

A correction is made for bias due to any tapering.

As in the description of G13CBF for univariate frequency window smoothing, the smoothed spectrum is returned as a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}, \quad l = 0, 1, \ldots, [L/2]$$

where [ ] denotes the integer part.

Its real part or co-spectrum $cf(\nu_l)$, and imaginary part or quadrature spectrum $qf(\nu_l)$ are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} \tilde{w}_k f_{xy}^*(\nu_l + \omega_k)$$

where the weights $\tilde{w}_k$ are similar to the weights w_k defined for G13CBF, but allow for an implicit alignment shift S between the series:

$$\tilde{w}_k = w_k \exp(-2\pi i S k/L).$$

It is recommended that S is chosen as the lag k at which the cross-covariances $c_{xy}(k)$ peak, so as to minimize bias.

If no smoothing is required, the integer M, which determines the frequency window width $\frac{2\pi}{M}$, should be set to n.

The bandwidth of the estimates will normally have been calculated in a previous call of G13CBF for estimating the univariate spectra of y_t and x_t.

4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden–Day

5 Arguments

1: NXY – INTEGER *Input*

On entry: n, the length of the time series x and y.

Constraint: NXY ≥ 1.

2: MTXY – INTEGER *Input*

On entry: whether the data is to be initially mean or trend corrected.

MTXY $= 0$
For no correction.

MTXY $= 1$
For mean correction.

MTXY $= 2$
For trend correction.

Constraint: $0 \leq$ MTXY ≤ 2.

3: PXY – REAL (KIND=nag_wp) *Input*

On entry: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.

A value of 0.0 implies no tapering.

Constraint: $0.0 \leq$ PXY ≤ 1.0.

4: MW – INTEGER *Input*

On entry: M, the frequency width of the smoothing window as $\frac{2\pi}{M}$.

A value of n implies that no smoothing is to be carried out.

Constraint: $1 \leq$ MW $\leq$ NXY.

5: ISH – INTEGER *Input*

On entry: S, the alignment shift between the x and y series. If x leads y, the shift is positive.

Constraint: $-$L $<$ ISH $<$ L.

6: PW – REAL (KIND=nag_wp) *Input*

On entry: p, the shape parameter of the trapezium frequency window.

A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.

If MW $=$ NXY (i.e., no smoothing is carried out) then PW is not used.

Constraint: if MW $\neq$ NXY, $0.0 \leq$ PW ≤ 1.0.

7: L – INTEGER *Input*

On entry: L, the frequency division of smoothed cross spectral estimates as $\frac{2\pi}{L}$.

Constraints:

L ≥ 1;
L must be a factor of KC.

8: KC – INTEGER *Input*

On entry: the dimension of the arrays XG and YG as declared in the (sub)program from which G13CDF is called. The order of the fast Fourier transform (FFT) used to calculate the spectral estimates.

Constraints:

KC $\geq 2 \times$ NXY;
KC must be a multiple of L.

9: XG(KC) – REAL (KIND=nag_wp) array *Input/Output*

On entry: the NXY data points of the x series.

On exit: the real parts of the NG cross spectral estimates in elements XG(1) to XG(NG), and XG(NG + 1) to XG(KC) contain 0.0. The y series leads the x series.

10: YG(KC) – REAL (KIND=nag_wp) array *Input/Output*

On entry: the NXY data points of the y series.

On exit: the imaginary parts of the NG cross spectral estimates in elements YG(1) to YG(NG), and YG(NG + 1) to YG(KC) contain 0.0. The y series leads the x series.

11: NG – INTEGER *Output*

On exit: the number of spectral estimates, $[L/2] + 1$, whose separate parts are held in XG and YG.

12: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, ISH = ⟨*value*⟩ and L = ⟨*value*⟩.
Constraint: |ISH| < L.

On entry, L = ⟨*value*⟩.
Constraint: L ≥ 1.

On entry, MTXY = ⟨*value*⟩.
Constraint: MTXY ≤ 2.

On entry, $\text{MTXY} = \langle value \rangle$.
Constraint: $\text{MTXY} \geq 0$.

On entry, $\text{MW} = \langle value \rangle$.
Constraint: $\text{MW} \geq 1$.

On entry, $\text{MW} = \langle value \rangle$ and $\text{NXY} = \langle value \rangle$.
Constraint: $\text{MW} \leq \text{NXY}$.

On entry, $\text{NXY} = \langle value \rangle$.
Constraint: $\text{NXY} \geq 1$.

On entry, $\text{PXY} = \langle value \rangle$, $\text{MW} = \langle value \rangle$ and $\text{NXY} = \langle value \rangle$.
Constraint: if $\text{PW} < 0.0$, $\text{MW} = \text{NXY}$.

On entry, $\text{PXY} = \langle value \rangle$, $\text{MW} = \langle value \rangle$ and $\text{NXY} = \langle value \rangle$.
Constraint: if $\text{PW} > 1.0$, $\text{MW} = \text{NXY}$.

On entry, $\text{PXY} = \langle value \rangle$.
Constraint: $\text{PXY} \geq 0.0$.

On entry, $\text{PXY} = \langle value \rangle$.
Constraint: $\text{PXY} \leq 1.0$.

$\text{IFAIL} = 2$

On entry, $\text{KC} = \langle value \rangle$ and $\text{L} = \langle value \rangle$.
Constraint: KC must be a multiple of L.

On entry, $\text{KC} = \langle value \rangle$ and $\text{NXY} = \langle value \rangle$.
Constraint: $\text{KC} \geq 2 \times \text{NXY}$.

$\text{IFAIL} = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

$\text{IFAIL} = -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

$\text{IFAIL} = -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Parallelism and Performance

G13CDF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G13CDF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

G13CDF carries out an FFT of length KC to calculate the sample cross spectrum. The time taken by the routine for this is approximately proportional to KC × log (KC) (but see routine document C06PAF for further details).

10 Example

This example reads two time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a $2\pi/16$ frequency width of smoothing window, a window shape parameter of 0.5 and an alignment shift of 3. It then calls G13CDF to calculate the smoothed sample cross spectrum and prints the results.

10.1 Program Text

```
    Program g13cdfe

!     G13CDF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: g13cdf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: pw, pxy
      Integer                          :: ifail, ish, j, kc, l, m, mtxy, mw,   &
                                          ng, nxy
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: xg(:), yg(:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: ceiling, log, real
!     .. Executable Statements ..
      Write (nout,*) 'G13CDF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Read in the problem size
      Read (nin,*) nxy, l

!     Read in control parameters
      Read (nin,*) mtxy, pxy, pw, mw, ish

!     Get a value for KC
      m = ceiling(log(2.0E0_nag_wp*real(nxy,kind=nag_wp)/real(l,              &
        kind=nag_wp))/log(2.0E0_nag_wp))
      kc = (2**m)*l
      Allocate (xg(kc),yg(kc))

!     Read in data
      Read (nin,*) xg(1:nxy)
      Read (nin,*) yg(1:nxy)

      ifail = 0
      Call g13cdf(nxy,mtxy,pxy,mw,ish,pw,l,kc,xg,yg,ng,ifail)

!     Display results
      Write (nout,*) '                      Returned sample spectrum'
      Write (nout,*)
      Write (nout,*)                                                          &
        '      Real  Imaginary       Real  Imaginary       Real  Imaginary'
      Write (nout,*)                                                          &
```

```
        '       part     part         part     part         part     part'
      Write (nout,99999)(j,xg(j),yg(j),j=1,ng)

99999 Format (1X,I3,F8.4,F9.4,I5,F8.4,F9.4,I5,F8.4,F9.4)
    End Program g13cdfe
```

10.2 Program Data

```
G13CDF Example Program Data
296 80                                                         :: NXY,L
1 0.1 0.5 16 3                                                 :: MTXY,PXY,PW,MW,ISH
 -0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
  0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
 -0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
  0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
  1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
  0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
  1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
  0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
  1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
 -1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
 -0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
 -0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
  1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
 -0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
 -2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
 -0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
  0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
  0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
  0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
  0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
  0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
 -1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
  0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
  0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
  0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
 -2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
 -0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
 -1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
 -1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
 -0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
 -1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
 -0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
  0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
  1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
 -0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
  0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
  0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262     :: End of XG
 53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4
 53.0 54.0 54.9 56.0 56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3
 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9 47.6 47.5 47.5 47.6
 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
 45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6
 51.5 51.6 51.2 50.5 50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7
 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3 56.6 56.0 55.4 55.4
 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
 51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8
 53.8 53.8 53.3 53.0 52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0
 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3 55.0 54.4 53.7 52.8
 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
 50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2
 53.0 52.8 52.3 51.9 51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3
 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9 54.6 53.5 52.4 52.1
 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
 53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0
 60.4 60.5 60.2 59.7 59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4
 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9 55.5 55.3 55.2 55.4
 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
```

```
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4
56.4 56.0 55.2 54.0 53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0
52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2 52.0 52.8 53.8 54.5
54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0                     :: End of YG
```

10.3 Program Results

```
G13CDF Example Program Results

                      Returned sample spectrum

      Real  Imaginary        Real  Imaginary        Real  Imaginary
      part     part          part     part          part     part
  1 -6.1563   0.0000    2 -5.5905  -2.0119    3 -3.2711  -2.7963
  4 -1.1803  -2.3264    5 -0.2061  -1.8132    6  0.3434  -1.1357
  7  0.6200  -0.7351    8  0.5967  -0.3449    9  0.4523  -0.0984
 10  0.2391   0.0177   11  0.1129   0.0402   12  0.0564   0.0523
 13  0.0134   0.0443   14 -0.0032   0.0299   15 -0.0057   0.0148
 16 -0.0057   0.0069   17 -0.0033   0.0038   18 -0.0011   0.0012
 19 -0.0004   0.0001   20 -0.0004   0.0002   21 -0.0003   0.0001
 22 -0.0001   0.0002   23 -0.0002   0.0003   24 -0.0002   0.0002
 25 -0.0002   0.0000   26 -0.0004   0.0000   27 -0.0002  -0.0002
 28 -0.0001  -0.0000   29 -0.0001   0.0002   30 -0.0001   0.0002
 31 -0.0002   0.0003   32 -0.0002   0.0001   33 -0.0001   0.0000
 34 -0.0000  -0.0000   35  0.0000  -0.0001   36  0.0001  -0.0001
 37  0.0001  -0.0001   38  0.0001  -0.0001   39  0.0000  -0.0001
 40  0.0000  -0.0001   41  0.0001   0.0000
```