

# NAG Library Routine Document

## F07MWF (ZHETRI)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07MWF (ZHETRI) computes the inverse of a complex Hermitian indefinite matrix  $A$ , where  $A$  has been factorized by F07MRF (ZHETRF).

### 2 Specification

```
SUBROUTINE F07MWF (UPLO, N, A, LDA, IPIV, WORK, INFO)
  INTEGER                N, LDA, IPIV(*), INFO
  COMPLEX (KIND=nag_wp) A(LDA,*), WORK(N)
  CHARACTER(1)          UPLO
```

The routine may be called by its LAPACK name *zhetri*.

### 3 Description

F07MWF (ZHETRI) is used to compute the inverse of a complex Hermitian indefinite matrix  $A$ , the routine must be preceded by a call to F07MRF (ZHETRF), which computes the Bunch–Kaufman factorization of  $A$ .

If  $UPLO = 'U'$ ,  $A = PUDU^H P^T$  and  $A^{-1}$  is computed by solving  $U^H P^T X P U = D^{-1}$  for  $X$ .

If  $UPLO = 'L'$ ,  $A = PLDL^H P^T$  and  $A^{-1}$  is computed by solving  $L^H P^T X P L = D^{-1}$  for  $X$ .

### 4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

### 5 Arguments

- 1: **UPLO** – CHARACTER(1) *Input*  
*On entry:* specifies how  $A$  has been factorized.  
**UPLO = 'U'**  
 $A = PUDU^H P^T$ , where  $U$  is upper triangular.  
**UPLO = 'L'**  
 $A = PLDL^H P^T$ , where  $L$  is lower triangular.  
*Constraint:* UPLO = 'U' or 'L'.
- 2: **N** – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .
- 3: **A(LDA,\*)** – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array  $A$  must be at least  $\max(1, N)$ .  
*On entry:* details of the factorization of  $A$ , as returned by F07MRF (ZHETRF).

*On exit:* the factorization is overwritten by the  $n$  by  $n$  Hermitian matrix  $A^{-1}$ .

If UPLO = 'U', the upper triangle of  $A^{-1}$  is stored in the upper triangular part of the array.

If UPLO = 'L', the lower triangle of  $A^{-1}$  is stored in the lower triangular part of the array.

4: LDA – INTEGER *Input*

*On entry:* the first dimension of the array A as declared in the (sub)program from which F07MWF (ZHETRI) is called.

*Constraint:*  $LDA \geq \max(1, N)$ .

5: IPIV(\*) – INTEGER array *Input*

**Note:** the dimension of the array IPIV must be at least  $\max(1, N)$ .

*On entry:* details of the interchanges and the block structure of  $D$ , as returned by F07MRF (ZHETRF).

6: WORK(N) – COMPLEX (KIND=nag\_wp) array *Workspace*

7: INFO – INTEGER *Output*

*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

$-999 < \text{INFO} < 0$

If  $\text{INFO} = -i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\text{INFO} = -999$

Dynamic memory allocation failed.

See Section 3.7 in *How to Use the NAG Library and its Documentation* for further information.

An explanatory message is output, and execution of the program is terminated.

$\text{INFO} > 0$

Element  $\langle \text{value} \rangle$  of the diagonal is exactly zero.  $D$  is singular and the inverse of  $A$  cannot be computed.

## 7 Accuracy

The computed inverse  $X$  satisfies a bound of the form

if UPLO = 'U',  $|DU^H P^T X P U - I| \leq c(n)\epsilon(|D||U^H|P^T|X|P|U| + |D||D^{-1}|)$ ;

if UPLO = 'L',  $|DL^H P^T X P L - I| \leq c(n)\epsilon(|D||L^H|P^T|X|P|L| + |D||D^{-1}|)$ ,

$c(n)$  is a modest linear function of  $n$ , and  $\epsilon$  is the *machine precision*.

## 8 Parallelism and Performance

F07MWF (ZHETRI) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of real floating-point operations is approximately  $\frac{8}{3}n^3$ .

The real analogue of this routine is F07MJF (DSYTRI).

## 10 Example

This example computes the inverse of the matrix  $A$ , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian indefinite and must first be factorized by F07MRF (ZHETRF).

### 10.1 Program Text

```

Program f07mwfe

!      F07MWF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zhetrf, zhetri
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Integer                     :: i, ifail, info, lda, lwork, n
      Character (1)               :: uplo
!      .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,,:), work(:)
      Integer, Allocatable         :: ipiv(:)
      Character (1)               :: clabs(1), rlabs(1)
!      .. Executable Statements ..
      Write (nout,*) 'F07MWF Example Program Results'
!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      lwork = 64*n
      Allocate (a(lda,n),work(lwork),ipiv(n))

!      Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If

!      Factorize A
!      The NAG name equivalent of zhetrf is f07mrf
      Call zhetrf(uplo,n,a,lda,ipiv,work,lwork,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

```

```

!      Compute inverse of A
!      The NAG name equivalent of zhetri is f07mwf
!      Call zhetri(uplo,n,a,lda,ipiv,work,info)

!      Print inverse

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
!      ifail = 0
!      Call x04dbf(uplo,'Nonunit',n,n,a,lda,'Bracketed','F7.4','Inverse',      &
!                  'Integer',rlabs,'Integer',clabs,80,0,ifail)

      Else
        Write (nout,*) 'The factor D is singular'
      End If

End Program f07mwfe

```

## 10.2 Program Data

F07MWF Example Program Data

```

4                                     :Value of N
'L'                                 :Value of UPLO
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A

```

## 10.3 Program Results

F07MWF Example Program Results

```

Inverse
          1          2          3          4
1 ( 0.0826, 0.0000)
2 (-0.0335, 0.0440) (-0.1408, 0.0000)
3 ( 0.0603,-0.0105) ( 0.0422,-0.0222) (-0.2007, 0.0000)
4 ( 0.2391,-0.0926) ( 0.0304, 0.0203) ( 0.0982,-0.0635) ( 0.0073,-0.0000)

```

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