

# NAG Library Routine Document

## F07CEF (DGTTRS)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07CEF (DGTTRS) computes the solution to a real system of linear equations  $AX = B$  or  $A^T X = B$ , where  $A$  is an  $n$  by  $n$  tridiagonal matrix and  $X$  and  $B$  are  $n$  by  $r$  matrices, using the  $LU$  factorization returned by F07CDF (DGTTRF).

### 2 Specification

```
SUBROUTINE F07CEF (TRANS, N, NRHS, DL, D, DU, DU2, IPIV, B, LDB, INFO)
  INTEGER          N, NRHS, IPIV(*), LDB, INFO
  REAL (KIND=nag_wp) DL(*), D(*), DU(*), DU2(*), B(LDB,*)
  CHARACTER(1)     TRANS
```

The routine may be called by its LAPACK name *dgtrrs*.

### 3 Description

F07CEF (DGTTRS) should be preceded by a call to F07CDF (DGTTRF), which uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix  $A$  as

$$A = PLU,$$

where  $P$  is a permutation matrix,  $L$  is unit lower triangular with at most one nonzero subdiagonal element in each column, and  $U$  is an upper triangular band matrix, with two superdiagonals. F07CEF (DGTTRS) then utilizes the factorization to solve the required equations.

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

### 5 Arguments

- 1: TRANS – CHARACTER(1) *Input*  
*On entry:* specifies the equations to be solved as follows:  
 TRANS = 'N'  
     Solve  $AX = B$  for  $X$ .  
 TRANS = 'T' or 'C'  
     Solve  $A^T X = B$  for  $X$ .  
*Constraint:* TRANS = 'N', 'T' or 'C'.
- 2: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .

- 3: NRHS – INTEGER *Input*  
*On entry:*  $r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .  
*Constraint:*  $\text{NRHS} \geq 0$ .
- 4: DL(\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array DL must be at least  $\max(1, N - 1)$ .  
*On entry:* must contain the  $(n - 1)$  multipliers that define the matrix  $L$  of the  $LU$  factorization of  $A$ .
- 5: D(\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array D must be at least  $\max(1, N)$ .  
*On entry:* must contain the  $n$  diagonal elements of the upper triangular matrix  $U$  from the  $LU$  factorization of  $A$ .
- 6: DU(\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array DU must be at least  $\max(1, N - 1)$ .  
*On entry:* must contain the  $(n - 1)$  elements of the first superdiagonal of  $U$ .
- 7: DU2(\*) – REAL (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array DU2 must be at least  $\max(1, N - 2)$ .  
*On entry:* must contain the  $(n - 2)$  elements of the second superdiagonal of  $U$ .
- 8: IPIV(\*) – INTEGER array *Input*  
**Note:** the dimension of the array IPIV must be at least  $\max(1, N)$ .  
*On entry:* must contain the  $n$  pivot indices that define the permutation matrix  $P$ . At the  $i$ th step, row  $i$  of the matrix was interchanged with row  $\text{IPIV}(i)$ , and  $\text{IPIV}(i)$  must always be either  $i$  or  $(i + 1)$ ,  $\text{IPIV}(i) = i$  indicating that a row interchange was not performed.
- 9: B(LDB, \*) – REAL (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array B must be at least  $\max(1, \text{NRHS})$ .  
*On entry:* the  $n$  by  $r$  matrix of right-hand sides  $B$ .  
*On exit:* the  $n$  by  $r$  solution matrix  $X$ .
- 10: LDB – INTEGER *Input*  
*On entry:* the first dimension of the array B as declared in the (sub)program from which F07CEF (DGTTRS) is called.  
*Constraint:*  $\text{LDB} \geq \max(1, N)$ .
- 11: INFO – INTEGER *Output*  
*On exit:*  $\text{INFO} = 0$  unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

$\text{INFO} < 0$

If  $\text{INFO} = -i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of this routine F07CGF (DGTCON) can be used to estimate the condition number of  $A$  and F07CHF (DGTRFS) can be used to obtain approximate error bounds.

## 8 Parallelism and Performance

F07CEF (DGTTRS) is not threaded in any implementation.

## 9 Further Comments

The total number of floating-point operations required to solve the equations  $AX = B$  or  $A^T X = B$  is proportional to  $nr$ .

The complex analogue of this routine is F07CSF (ZGTTRS).

## 10 Example

This example solves the equations

$$AX = B,$$

where  $A$  is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2.7 & 6.6 \\ -0.5 & 10.8 \\ 2.6 & -3.2 \\ 0.6 & -11.2 \\ 2.7 & 19.1 \end{pmatrix}.$$

### 10.1 Program Text

Program f07cefe

```
!      F07CEF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: dgtrrf, dgtrrs, nag_wp, x04caf
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Integer                     :: i, ifail, info, ldb, n, nrhs
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: b(:, :), d(:, :), dl(:, :), du(:, :), du2(:, :),
      Integer, Allocatable         :: ipiv(:)
!      .. Executable Statements ..
```

```

      Write (nout,*) 'F07CEF Example Program Results'
      Write (nout,*)
      Flush (nout)
!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs
      ldb = n
      Allocate (b(ldb,nrhs),d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

!      Read the tridiagonal matrix A from data file

      Read (nin,*) du(1:n-1)
      Read (nin,*) d(1:n)
      Read (nin,*) dl(1:n-1)

!      Read the right hand matrix B
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!      Factorize the tridiagonal matrix A
!      The NAG name equivalent of dgttrf is f07cdf
      Call dgttrf(n,dl,d,du,du2,ipiv,info)

      If (info==0) Then

!          Solve the equations AX = B
!          The NAG name equivalent of dgttrs is f07cef
          Call dgttrs('No transpose',n,nrhs,dl,d,du,du2,ipiv,b,ldb,info)

!          Print the solution

!          ifail: behaviour on error exit
!          =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
          ifail = 0
          Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

      Else
          Write (nout,99999) 'The (', info, ',', info, ')',
            ' element of the factor U is zero'
      End If

99999 Format (1X,A,I3,A,I3,A,A)
      End Program f07cefe

```

## 10.2 Program Data

```

F07CEF Example Program Data
  5      2      :Values of N and NRHS
      2.1 -1.0  1.9  8.0
  3.0  2.3 -5.0 -0.9  7.1
  3.4  3.6  7.0 -6.0      :End of matrix A
  2.7  6.6
-0.5 10.8
  2.6 -3.2
  0.6 -11.2
  2.7 19.1      :End of matrix B

```

## 10.3 Program Results

F07CEF Example Program Results

```

Solution(s)
      1      2
  1    -4.0000  5.0000
  2     7.0000 -4.0000
  3     3.0000 -3.0000
  4    -4.0000 -2.0000
  5    -3.0000  1.0000

```

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