

# NAG Library Routine Document

## F07CDF (DGTTRF)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07CDF (DGTTRF) computes the  $LU$  factorization of a real  $n$  by  $n$  tridiagonal matrix  $A$ .

### 2 Specification

```
SUBROUTINE F07CDF (N, DL, D, DU, DU2, IPIV, INFO)
  INTEGER          N, IPIV(N), INFO
  REAL (KIND=nag_wp) DL(*), D(*), DU(*), DU2(N-2)
```

The routine may be called by its LAPACK name *dgtrf*.

### 3 Description

F07CDF (DGTTRF) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix  $A$  as

$$A = PLU,$$

where  $P$  is a permutation matrix,  $L$  is unit lower triangular with at most one nonzero subdiagonal element in each column, and  $U$  is an upper triangular band matrix, with two superdiagonals.

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

### 5 Arguments

- 1: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .
- 2: DL(\*) – REAL (KIND=nag\_wp) array *Input/Output*  
**Note:** the dimension of the array DL must be at least  $\max(1, N - 1)$ .  
*On entry:* must contain the  $(n - 1)$  subdiagonal elements of the matrix  $A$ .  
*On exit:* is overwritten by the  $(n - 1)$  multipliers that define the matrix  $L$  of the  $LU$  factorization of  $A$ .
- 3: D(\*) – REAL (KIND=nag\_wp) array *Input/Output*  
**Note:** the dimension of the array D must be at least  $\max(1, N)$ .  
*On entry:* must contain the  $n$  diagonal elements of the matrix  $A$ .  
*On exit:* is overwritten by the  $n$  diagonal elements of the upper triangular matrix  $U$  from the  $LU$  factorization of  $A$ .

- 4: DU(\*) – REAL (KIND=nag\_wp) array *Input/Output*  
**Note:** the dimension of the array DU must be at least  $\max(1, N - 1)$ .  
*On entry:* must contain the  $(n - 1)$  superdiagonal elements of the matrix  $A$ .  
*On exit:* is overwritten by the  $(n - 1)$  elements of the first superdiagonal of  $U$ .
- 5: DU2(N - 2) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* contains the  $(n - 2)$  elements of the second superdiagonal of  $U$ .
- 6: IPIV(N) – INTEGER array *Output*  
*On exit:* contains the  $n$  pivot indices that define the permutation matrix  $P$ . At the  $i$ th step, row  $i$  of the matrix was interchanged with row IPIV( $i$ ). IPIV( $i$ ) will always be either  $i$  or  $(i + 1)$ , IPIV( $i$ ) =  $i$  indicating that a row interchange was not performed.
- 7: INFO – INTEGER *Output*  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

Element  $\langle value \rangle$  of the diagonal is exactly zero. The factorization has been completed, but the factor  $U$  is exactly singular, and division by zero will occur if it is used to solve a system of equations.

## 7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = PLU,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and  $\epsilon$  is the *machine precision*.

Following the use of this routine, F07CEF (DGTTRS) can be used to solve systems of equations  $AX = B$  or  $A^T X = B$ , and F07CGF (DGTCON) can be used to estimate the condition number of  $A$ .

## 8 Parallelism and Performance

F07CDF (DGTTRF) is not threaded in any implementation.

## 9 Further Comments

The total number of floating-point operations required to factorize the matrix  $A$  is proportional to  $n$ .

The complex analogue of this routine is F07CRF (ZGTTRF).

## 10 Example

This example factorizes the tridiagonal matrix  $A$  given by

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix}.$$

### 10.1 Program Text

```

Program f07cdf

!      F07CDF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: dgttrf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Integer                     :: info, n
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: d(:), dl(:), du(:), du2(:)
      Integer, Allocatable        :: ipiv(:)
!      .. Executable Statements ..
      Write (nout,*) 'F07CDF Example Program Results'
      Write (nout,*)
!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

!      Read the tridiagonal matrix A from data file

      Read (nin,*) du(1:n-1)
      Read (nin,*) d(1:n)
      Read (nin,*) dl(1:n-1)

!      Factorize the tridiagonal matrix A
!      The NAG name equivalent of dgttrf is f06cdf
      Call dgttrf(n,dl,d,du,du2,ipiv,info)

      If (info>0) Then
         Write (nout,99999) 'The (', info, ',', info, ')',
            ' element of the factor U is zero'
      End If

!      Print details of the factorization

      Write (nout,*) 'Details of factorization'
      Write (nout,*)
      Write (nout,*) ' Second superdiagonal of U'
      Write (nout,99998) du2(1:n-2)
      Write (nout,*)
      Write (nout,*) ' First superdiagonal of U'
      Write (nout,99998) du(1:n-1)
      Write (nout,*)
      Write (nout,*) ' Main diagonal of U'
      Write (nout,99998) d(1:n)
      Write (nout,*)
      Write (nout,*) ' Multipliers'
      Write (nout,99998) dl(1:n-1)
      Write (nout,*)

```

```

      Write (nout,*) ' Vector of interchanges'
      Write (nout,99997) ipiv(1:n)

99999 Format (1X,A,I3,A,I3,A,A)
99998 Format (1X,8F9.4)
99997 Format (1X,5I9)
      End Program f07cdfe

```

## 10.2 Program Data

```

F07CDF Example Program Data
  5                               :Value of N
      2.1  -1.0   1.9   8.0
  3.0   2.3  -5.0  -0.9   7.1
  3.4   3.6   7.0  -6.0       :End of matrix A

```

## 10.3 Program Results

F07CDF Example Program Results

Details of factorization

Second superdiagonal of U  
 -1.0000 1.9000 8.0000

First superdiagonal of U  
 2.3000 -5.0000 -0.9000 7.1000

Main diagonal of U  
 3.4000 3.6000 7.0000 -6.0000 -1.0154

Multipliers  
 0.8824 0.0196 0.1401 -0.0148

Vector of interchanges  
 2 3 4 5 5

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