

NAG Library Routine Document

F07BEF (DGBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BEF (DGBTRS) solves a real band system of linear equations with multiple right-hand sides,

$$AX = B \quad \text{or} \quad A^T X = B,$$

where A has been factorized by F07BDF (DGBTRF).

2 Specification

```
SUBROUTINE F07BEF (TRANS, N, KL, KU, NRHS, AB, LDAB, IPIV, B, LDB, INFO)
INTEGER          N, KL, KU, NRHS, LDAB, IPIV(*), LDB, INFO
REAL (KIND=nag_wp) AB(LDAB,*), B(LDB,*)
CHARACTER(1)     TRANS
```

The routine may be called by its LAPACK name *dgbtrs*.

3 Description

F07BEF (DGBTRS) is used to solve a real band system of linear equations $AX = B$ or $A^T X = B$, the routine must be preceded by a call to F07BDF (DGBTRF) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If $TRANS = 'N'$, the solution is computed by solving $PLY = B$ and then $UX = Y$.

If $TRANS = 'T'$ or $'C'$, the solution is computed by solving $U^T Y = B$ and then $L^T P^T X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

- 1: TRANS – CHARACTER(1) *Input*
On entry: indicates the form of the equations.
 TRANS = 'N'
 $AX = B$ is solved for X .
 TRANS = 'T' or 'C'
 $A^T X = B$ is solved for X .
Constraint: TRANS = 'N', 'T' or 'C'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.

- 3: KL – INTEGER *Input*
On entry: k_l , the number of subdiagonals within the band of the matrix A .
Constraint: $KL \geq 0$.
- 4: KU – INTEGER *Input*
On entry: k_u , the number of superdiagonals within the band of the matrix A .
Constraint: $KU \geq 0$.
- 5: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: $NRHS \geq 0$.
- 6: AB(LDAB,*) – REAL (KIND=nag_wp) array *Input*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the LU factorization of A , as returned by F07BDF (DGBTRF).
- 7: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07BEF (DGBTRS) is called.
Constraint: $LDAB \geq 2 \times KL + KU + 1$.
- 8: IPIV(*) – INTEGER array *Input*
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: the pivot indices, as returned by F07BDF (DGBTRF).
- 9: B(LDB,*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, NRHS)$.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 10: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07BEF (DGBTRS) is called.
Constraint: $LDB \geq \max(1, N)$.
- 11: INFO – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

$INFO < 0$

If $INFO = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the **machine precision**. This assumes $k \ll n$.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(k) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \text{cond}(A) = \| |A^{-1}| |A| \|_\infty \leq \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$, and $\text{cond}(A^T)$ can be much larger (or smaller) than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling F07BHF (DGBRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07BGF (DGBCON) with NORM = 'I'.

8 Parallelism and Performance

F07BEF (DGBTRS) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07BEF (DGBTRS) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $2n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This routine may be followed by a call to F07BHF (DGBRFS) to refine the solution and return an error estimate.

The complex analogue of this routine is F07BSF (ZGBTRS).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4.42 & -36.01 \\ 27.13 & -31.67 \\ -6.14 & -1.16 \\ 10.50 & -25.82 \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by F07BDF (DGBTRF).

10.1 Program Text

Program f07befe

```

!      F07BEF Example Program Text
!
!      Mark 26 Release. NAG Copyright 2016.
!
!      .. Use Statements ..
!      Use nag_library, Only: dgbtrf, dgbtrs, nag_wp, x04caf
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      Character (1), Parameter   :: trans = 'N'
!      .. Local Scalars ..
!      Integer                    :: i, ifail, info, j, k, kl, ku, ldab, &
!                                   ldb, n, nrhs
!      .. Local Arrays ..
!      Real (Kind=nag_wp), Allocatable :: ab(:,,:), b(:,:)
!      Integer, Allocatable           :: ipiv(:)
!      .. Intrinsic Procedures ..
!      Intrinsic                     :: max, min
!      .. Executable Statements ..
!      Write (nout,*) 'F07BEF Example Program Results'
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n, nrhs, kl, ku
!      ldab = 2*kl + ku + 1
!      ldb = n
!      Allocate (ab(ldab,n),b(ldb,nrhs),ipiv(n))
!
!      Read A and B from data file
!
!      k = kl + ku + 1
!      Read (nin,*)((ab(k+i-j,j),j=max(i-kl,1),min(i+ku,n)),i=1,n)
!      Read (nin,*)(b(i,1:nrhs),i=1,n)
!
!      Factorize A
!      The NAG name equivalent of dgbtrf is f07bdf
!      Call dgbtrf(n,n,kl,ku,ab,ldab,ipiv,info)
!
!      Write (nout,*)
!      Flush (nout)
!      If (info==0) Then
!
!          Compute solution
!          The NAG name equivalent of dgbtrs is f07bef
!          Call dgbtrs(trans,n,kl,ku,nrhs,ab,ldab,ipiv,b,ldb,info)
!
!          Print solution
!
!          ifail: behaviour on error exit
!                  =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
!          ifail = 0
!          Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)
!
!      Else
!          Write (nout,*) 'The factor U is singular'
!      End If
!
!      End Program f07befe

```

10.2 Program Data

```
F07BEF Example Program Data
  4  2  1  2          :Values of N, NRHS, KL and KU
-0.23  2.54 -3.66
-6.98  2.46 -2.73 -2.13
        2.56  2.46  4.07
        -4.78 -3.82    :End of matrix A
  4.42 -36.01
 27.13 -31.67
-6.14  -1.16
 10.50 -25.82          :End of matrix B
```

10.3 Program Results

F07BEF Example Program Results

```
Solution(s)
           1           2
1      -2.0000      1.0000
2       3.0000     -4.0000
3       1.0000      7.0000
4      -4.0000     -2.0000
```
