

# NAG Library Routine Document

## F06TPF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

F06TPF performs a  $QR$  factorization (as a sequence of plane rotations) of a complex upper triangular matrix that has been modified by a rank-1 update.

### 2 Specification

```
SUBROUTINE F06TPF (N, ALPHA, X, INCX, Y, INCY, A, LDA, C, S)
INTEGER          N, INCX, INCY, LDA
REAL (KIND=nag_wp) C(N-1)
COMPLEX (KIND=nag_wp) ALPHA, X(*), Y(*), A(LDA,*), S(N)
```

### 3 Description

F06TPF performs a  $QR$  factorization of an upper triangular matrix which has been modified by a rank-1 update:

$$\alpha xy^T + U = QR$$

where  $U$  and  $R$  are  $n$  by  $n$  complex upper triangular matrices with real diagonal elements,  $x$  and  $y$  are  $n$ -element complex vectors,  $\alpha$  is a complex scalar, and  $Q$  is an  $n$  by  $n$  complex unitary matrix.

$Q$  is formed as the product of two sequences of plane rotations and a unitary diagonal matrix  $D$ :

$$Q^H = DQ_{n-1} \cdots Q_2Q_1P_1P_2 \cdots P_{n-1}$$

where

$P_k$  is a rotation in the  $(k, n)$  plane, chosen to annihilate  $x_k$ : thus  $Px = \beta e_n$ , where  $P = P_1P_2 \cdots P_{n-1}$  and  $e_n$  is the last column of the unit matrix;

$Q_k$  is a rotation in the  $(k, n)$  plane, chosen to annihilate the  $(n, k)$  element of  $(\alpha\beta e_n y^T + PU)$ , and thus restore it to upper triangular form;

$D = \text{diag}(1, \dots, 1, d_n)$ , with  $d_n$  chosen to make  $r_{nn}$  real;  $|d_n| = 1$ .

The 2 by 2 plane rotation part of  $P_k$  or  $Q_k$  has the form

$$\begin{pmatrix} c_k & \bar{s}_k \\ -s_k & c_k \end{pmatrix}$$

with  $c_k$  real. The tangents of the rotations  $P_k$  are returned in the array X; the cosines and sines of these rotations can be recovered by calling F06BCF. The cosines and sines of the rotations  $Q_k$  are returned directly in the arrays C and S.

### 4 References

None.

## 5 Arguments

- 1: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrices  $U$  and  $R$ .  
*Constraint:*  $N \geq 0$ .
  
- 2: ALPHA – COMPLEX (KIND=nag\_wp) *Input*  
*On entry:* the scalar  $\alpha$ .
  
- 3: X(\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the dimension of the array X must be at least  $\max(1, 1 + (N - 1) \times \text{INCX})$ .  
*On entry:* the  $n$ -element vector  $x$ .  $x_i$  must be stored in  $X(1 + (i - 1) \times \text{INCX})$ , for  $i = 1, 2, \dots, N$ .  
 Intermediate elements of X are not referenced.  
*On exit:* the referenced elements are overwritten by details of the sequence of plane rotations.
  
- 4: INCX – INTEGER *Input*  
*On entry:* the increment in the subscripts of X between successive elements of  $x$ .  
*Constraint:*  $\text{INCX} > 0$ .
  
- 5: Y(\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array Y must be at least  $\max(1, 1 + (N - 1) \times \text{INCY})$ .  
*On entry:* the  $n$ -element vector  $y$ .  $y_i$  must be stored in  $Y(1 + (i - 1) \times \text{INCY})$ , for  $i = 1, 2, \dots, N$ .  
 Intermediate elements of Y are not referenced.
  
- 6: INCY – INTEGER *Input*  
*On entry:* the increment in the subscripts of Y between successive elements of  $y$ .  
*Constraint:*  $\text{INCY} > 0$ .
  
- 7: A(LDA,\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array A must be at least N.  
*On entry:* the  $n$  by  $n$  upper triangular matrix  $U$ . The imaginary parts of the diagonal elements must be zero.  
*On exit:* the upper triangular matrix  $R$ . The imaginary parts of the diagonal elements must be zero.
  
- 8: LDA – INTEGER *Input*  
*On entry:* the first dimension of the array A as declared in the (sub)program from which F06TPF is called.  
*Constraint:*  $\text{LDA} \geq \max(1, N)$ .
  
- 9: C(N - 1) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* the cosines of the rotations  $Q_k$ , for  $k = 1, 2, \dots, n - 1$ .
  
- 10: S(N) – COMPLEX (KIND=nag\_wp) array *Output*  
*On exit:* the sines of the rotations  $Q_k$ , for  $k = 1, 2, \dots, n - 1$ ; S( $n$ ) holds  $d_n$ , the  $n$ th diagonal element of  $D$ .

## 6 Error Indicators and Warnings

None.

## 7 Accuracy

Not applicable.

## 8 Parallelism and Performance

F06TPF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

None.

## 10 Example

None.

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