

# NAG Library Routine Document

## D03NEF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

D03NEF computes average values of a continuous function of time over the remaining life of an option. It is used together with D03NDF to value options with time-dependent arguments.

### 2 Specification

```
SUBROUTINE D03NEF (T0, TMAT, NTD, TD, PHID, PHIAV, WORK, LWORK, IFAIL)
  INTEGER          NTD, LWORK, IFAIL
  REAL (KIND=nag_wp) T0, TMAT, TD(NTD), PHID(NTD), PHIAV(3), WORK(LWORK)
```

### 3 Description

D03NEF computes the quantities

$$\phi(t_0), \quad \hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left( \frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}$$

from a given set of values PHID of a continuous time-dependent function  $\phi(t)$  at a set of discrete points TD, where  $t_0$  is the current time and  $T$  is the maturity time. Thus  $\hat{\phi}$  and  $\bar{\phi}$  are first and second order averages of  $\phi$  over the remaining life of an option.

The routine may be used in conjunction with D03NDF in order to value an option in the case where the risk-free interest rate  $r$ , the continuous dividend  $q$ , or the stock volatility  $\sigma$  is time-dependent and is described by values at a set of discrete times (see Section 9.2). This is illustrated in Section 10.

### 4 References

None.

### 5 Arguments

- |    |  |              |
|----|--|--------------|
| 1: | T0 – REAL (KIND=nag_wp)<br><i>On entry:</i> the current time $t_0$ .<br><i>Constraint:</i> $\text{TD}(1) \leq \text{T0} \leq \text{TD}(\text{NTD})$ .    | <i>Input</i> |
| 2: | TMAT – REAL (KIND=nag_wp)<br><i>On entry:</i> the maturity time $T$ .<br><i>Constraint:</i> $\text{TD}(1) \leq \text{TMAT} \leq \text{TD}(\text{NTD})$ . | <i>Input</i> |
| 3: | NTD – INTEGER<br><i>On entry:</i> the number of discrete times at which $\phi$ is given.<br><i>Constraint:</i> $\text{NTD} \geq 2$ .                     | <i>Input</i> |

- 4: TD(NTD) – REAL (KIND=nag\_wp) array *Input*  
*On entry:* the discrete times at which  $\phi$  is specified.  
*Constraint:*  $TD(1) < TD(2) < \dots < TD(NTD)$ .
- 5: PHID(NTD) – REAL (KIND=nag\_wp) array *Input*  
*On entry:* PHID( $i$ ) must contain the value of  $\phi$  at time TD( $i$ ), for  $i = 1, 2, \dots, NTD$ .
- 6: PHIIV(3) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* PHIIV(1) contains the value of  $\phi$  interpolated to  $t_0$ , PHIIV(2) contains the first-order average  $\hat{\phi}$  and PHIIV(3) contains the second-order average  $\bar{\phi}$ , where:
- $$\hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left( \frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}.$$
- 7: WORK(LWORK) – REAL (KIND=nag\_wp) array *Workspace*  
8: LWORK – INTEGER *Input*  
*On entry:* the dimension of the array WORK as declared in the (sub)program from which D03NEF is called.  
*Constraint:*  $LWORK \geq 9 \times NTD + 24$ .
- 9: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.  
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, T0 lies outside the range [TD(1), TD(NTD)],  
or T0 lies outside the range [TD(1), TD(NTD)],  
or  $NTD < 2$ ,  
or TD badly ordered,  
or  $LWORK < 9 \times NTD + 24$ .

IFAIL = 2

Unexpected failure in internal call to E01BAF or E02BBF.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

If  $\phi \in C^4[t_0, T]$  then the error in the approximation of  $\phi(t_0)$  and  $\hat{\phi}$  is  $O(H^4)$ , where  $H = \max_i (T(i+1) - T(i))$ , for  $i = 1, 2, \dots, \text{NTD} - 1$ . The approximation is exact for polynomials of degree up to 3.

The third quantity  $\bar{\phi}$  is  $O(H^2)$ , and exact for linear functions.

## 8 Parallelism and Performance

D03NEF is not threaded in any implementation.

## 9 Further Comments

### 9.1 Timing

The time taken is proportional to NTD.

### 9.2 Use with D03NDF

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate  $r$  is a known function of time, and is represented as a set of values at discrete times. A call to D03NEF providing these values in PHID produces an output array PHIIV suitable for use as the argument R in a subsequent call to D03NDF.

Time-dependent values of the continuous dividend  $Q$  and the volatility  $\sigma$  may be handled in the same way.

### 9.3 Algorithmic Details

The NTD data points are fitted with a cubic B-spline using the routine E01BAF. Evaluation is then performed using E02BBF, and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of  $T = t_0$  is handled by interpolating  $\phi$  at that point.

## 10 Example

This example demonstrates the use of the routine in conjunction with D03NDF to solve the Black–Scholes equation for valuation of a 5-month American call option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate varies linearly with time and the stock volatility has a quadratic variation. Since these functions are integrated exactly by D03NEF the solution of the Black–Scholes equation by D03NDF is also exact.

The option is valued at a range of times and stock prices.

## 10.1 Program Text

```

!   D03NEF Example Program Text
!   Mark 26 Release. NAG Copyright 2016.

Module d03nefe_mod

!   D03NEF Example Program Module:
!       Parameters and User-defined Routines

!   .. Use Statements ..
Use nag_library, Only: nag_wp
!   .. Implicit None Statement ..
Implicit None
!   .. Accessibility Statements ..
Private
Public
!   .. Parameters ..
Integer, Parameter, Public      :: nin = 5, nout = 6
Contains
Subroutine print_greek(ns,nt,tmat,s,t,gname,greek)

!   .. Scalar Arguments ..
Real (Kind=nag_wp), Intent (In) :: tmat
Integer, Intent (In)           :: ns, nt
Character (*), Intent (In)     :: gname
!   .. Array Arguments ..
Real (Kind=nag_wp), Intent (In) :: greek(ns,nt), s(ns), t(nt)
!   .. Local Scalars ..
Integer                          :: i, j
!   .. Intrinsic Procedures ..
Intrinsic                       :: len
!   .. Executable Statements ..
Write (nout,*)
Write (nout,*) gname
Write (nout,*) ('-',i=1,len(gname))
Write (nout,*) ' Stock Price | Time to Maturity (months)'
Write (nout,99999) '|', (12.0_nag_wp*(tmat-t(i)),i=1,nt)
Write (nout,*) ' -----', ('-----',i=1,nt)
Do i = 1, ns
    Write (nout,99998) s(i), '|', (greek(i,j),j=1,nt)
End Do

Return

99999  Format (16X,A,1X,12(1P,E12.4))
99998  Format (1X,1P,E12.4,3X,A,1X,12(1P,E12.4))
End Subroutine print_greek
End Module d03nefe_mod

Program d03nefe

!   D03NEF Example Main Program

!   .. Use Statements ..
Use nag_library, Only: d03ndf, d03nef, nag_wp
Use d03nefe_mod, Only: nin, nout, print_greek
!   .. Implicit None Statement ..
Implicit None
!   .. Parameters ..
Logical, Parameter              :: gprnt(5) = .True.
!   .. Local Scalars ..
Real (Kind=nag_wp)              :: ds, dt, tmat, x
Integer                          :: i, ifail, j, kopt, lwork, ns, nt, &
                                ntd
!   .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: delta(:,,:), f(:,,:), gamma(:,,:), &
                                lambda(:,,:), rd(:,,:), rho(:,,:), s(:,) &
                                sigd(:,), t(:,), td(:,), theta(:,,:), &
                                work(:)
Real (Kind=nag_wp)              :: q(3), ra(3), siga(3)

```

```

      Logical                                :: tdpair(3)
!      .. Intrinsic Procedures ..
      Intrinsic                              :: real
!      .. Executable Statements ..
      Write (nout,*) 'D03NEF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) ns, nt, ntd
      lwork = 9*ntd + 24

      Allocate (delta(ns,nt),f(ns,nt),gamma(ns,nt),lambda(ns,nt),rd(ntd),      &
               rho(ns,nt),s(ns),sigd(ntd),t(nt),td(ntd),theta(ns,nt),work(lwork))

!      Read problem parameters

      Read (nin,*) kopt
      Read (nin,*) x
      Read (nin,*) tmat
      Read (nin,*) s(1), s(ns)
      Read (nin,*) t(1), t(nt)
      Read (nin,*) td(1:ntd)
      Read (nin,*) rd(1:ntd)
      Read (nin,*) sigd(1:ntd)
      Read (nin,*) tdpair(1:3)
      Read (nin,*) q(1)

      If (ns<2) Then
        Write (nout,*) 'NS invalid.'
      Else If (nt<2) Then
        Write (nout,*) 'NT invalid.'
      Else

        ds = (s(ns)-s(1))/real(ns-1,kind=nag_wp)
        dt = (t(nt)-t(1))/real(nt-1,kind=nag_wp)

!      Loop over times

        Do j = 1, nt

          t(j) = t(1) + real(j-1,kind=nag_wp)*dt

!      Find average values of r and sigma
          ifail = 0
          Call d03nef(t(j),tmat,ntd,td,rd,ra,work,lwork,ifail)

          ifail = 0
          Call d03nef(t(j),tmat,ntd,td,sigd,siga,work,lwork,ifail)

!      Loop over stock prices

          Do i = 1, ns

            s(i) = s(1) + real(i-1,kind=nag_wp)*ds

!      Evaluate analytic solution of Black-Scholes equation
            ifail = 0
            Call d03ndf(kopt,x,s(i),t(j),tmat,tdpair,ra,q,siga,f(i,j),      &
                      theta(i,j),delta(i,j),gamma(i,j),lambda(i,j),rho(i,j),ifail)

          End Do
        End Do

!      Output option values and possibly Greeks.

      Call print_greek(ns,nt,tmat,s,t,'Option Values',f)

      If (gprnt(1)) Then
        Call print_greek(ns,nt,tmat,s,t,'Theta',theta)
      End If

```

```

      If (gprnt(2)) Then
        Call print_greek(ns,nt,tmat,s,t,'Delta',delta)
      End If
      If (gprnt(3)) Then
        Call print_greek(ns,nt,tmat,s,t,'Gamma',gamma)
      End If
      If (gprnt(4)) Then
        Call print_greek(ns,nt,tmat,s,t,'Lambda',lambda)
      End If
      If (gprnt(5)) Then
        Call print_greek(ns,nt,tmat,s,t,'Rho',rho)
      End If

    End If

  End Program d03nefe

```

## 10.2 Program Data

D03NEF Example Program Data

```

21 4 6      : ns, nt, ntd
2           : kopt
50.         : x
0.4166667   : tmat
0.0 100.    : s(1), s(ns)
0.0 0.125   : t(1), t(nt)
0.00 0.10 0.20
0.30 0.40 0.50      : td
0.10 0.11 0.12
0.13 0.14 0.15      : rd
0.30 0.46 0.54
0.54 0.46 0.30      : sigd
.TRUE. .FALSE. .TRUE. : tdparr
0.0           : q(1)

```

## 10.3 Program Results

D03NEF Example Program Results

Option Values

Stock Price	Time to Maturity (months)				
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00	
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	
5.0000E+00	3.3671E-13	7.7404E-14	7.3210E-15	2.0179E-16	
1.0000E+01	5.2088E-07	2.4281E-07	7.2216E-08	1.1540E-08	
1.5000E+01	2.6607E-04	1.6753E-04	8.0943E-05	2.7179E-05	
2.0000E+01	8.9697E-03	6.6505E-03	4.1780E-03	2.0942E-03	
2.5000E+01	8.3647E-02	6.8467E-02	5.0375E-02	3.2105E-02	
3.0000E+01	3.8221E-01	3.3331E-01	2.7117E-01	2.0119E-01	
3.5000E+01	1.1298E+00	1.0275E+00	8.9292E-01	7.3146E-01	
4.0000E+01	2.5164E+00	2.3541E+00	2.1380E+00	1.8699E+00	
4.5000E+01	4.6249E+00	4.4110E+00	4.1267E+00	3.7700E+00	
5.0000E+01	7.4287E+00	7.1797E+00	6.8531E+00	6.4449E+00	
5.5000E+01	1.0830E+01	1.0564E+01	1.0221E+01	9.7996E+00	
6.0000E+01	1.4707E+01	1.4436E+01	1.4097E+01	1.3689E+01	
6.5000E+01	1.8937E+01	1.8671E+01	1.8348E+01	1.7968E+01	
7.0000E+01	2.3421E+01	2.3164E+01	2.2860E+01	2.2514E+01	
7.5000E+01	2.8080E+01	2.7833E+01	2.7550E+01	2.7234E+01	
8.0000E+01	3.2857E+01	3.2620E+01	3.2354E+01	3.2064E+01	
8.5000E+01	3.7713E+01	3.7484E+01	3.7233E+01	3.6963E+01	
9.0000E+01	4.2620E+01	4.2398E+01	4.2158E+01	4.1904E+01	
9.5000E+01	4.7561E+01	4.7344E+01	4.7112E+01	4.6868E+01	
1.0000E+02	5.2523E+01	5.2310E+01	5.2084E+01	5.1848E+01	

Theta

Stock Price	Time to Maturity (months)
-------------	---------------------------

	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	-8.9082E-12	-3.4507E-12	-5.0884E-13	-2.1236E-14
1.0000E+01	-7.2097E-06	-5.5915E-06	-2.5721E-06	-6.1830E-07
1.5000E+01	-2.2499E-03	-2.3259E-03	-1.7227E-03	-8.6349E-04
2.0000E+01	-4.9483E-02	-5.9355E-02	-5.6562E-02	-4.1921E-02
2.5000E+01	-3.1200E-01	-4.0620E-01	-4.4765E-01	-4.1683E-01
3.0000E+01	-9.8578E-01	-1.3408E+00	-1.6092E+00	-1.7186E+00
3.5000E+01	-2.0479E+00	-2.8395E+00	-3.5745E+00	-4.1390E+00
4.0000E+01	-3.2501E+00	-4.5165E+00	-5.8147E+00	-7.0323E+00
4.5000E+01	-4.3144E+00	-5.9349E+00	-7.6762E+00	-9.4488E+00
5.0000E+01	-5.0802E+00	-6.8543E+00	-8.7919E+00	-1.0815E+01
5.5000E+01	-5.5225E+00	-7.2603E+00	-9.1500E+00	-1.1104E+01
6.0000E+01	-5.7006E+00	-7.2722E+00	-8.9491E+00	-1.0625E+01
6.5000E+01	-5.7014E+00	-7.0446E+00	-8.4366E+00	-9.7565E+00
7.0000E+01	-5.6037E+00	-6.7093E+00	-7.8142E+00	-8.7951E+00
7.5000E+01	-5.4653E+00	-6.3555E+00	-7.2107E+00	-7.9170E+00
8.0000E+01	-5.3218E+00	-6.0329E+00	-6.6903E+00	-7.1974E+00
8.5000E+01	-5.1920E+00	-5.7627E+00	-6.2736E+00	-6.6481E+00
9.0000E+01	-5.0833E+00	-5.5487E+00	-5.9563E+00	-6.2492E+00
9.5000E+01	-4.9969E+00	-5.3857E+00	-5.7234E+00	-5.9700E+00
1.0000E+02	-4.9306E+00	-5.2651E+00	-5.5570E+00	-5.7797E+00

Delta

Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	1.6086E-12	3.8832E-13	3.9572E-14	1.2111E-15
1.0000E+01	8.9933E-07	4.3972E-07	1.4063E-07	2.4884E-08
1.5000E+01	2.3975E-04	1.5810E-04	8.1943E-05	3.0366E-05
2.0000E+01	4.9150E-03	3.8095E-03	2.5596E-03	1.4100E-03
2.5000E+01	3.0345E-02	2.5906E-02	2.0311E-02	1.4153E-02
3.0000E+01	9.6991E-02	8.7980E-02	7.5946E-02	6.1231E-02
3.5000E+01	2.0863E-01	1.9675E-01	1.8053E-01	1.5957E-01
4.0000E+01	3.4875E-01	3.3719E-01	3.2158E-01	3.0109E-01
4.5000E+01	4.9361E-01	4.8480E-01	4.7356E-01	4.5924E-01
5.0000E+01	6.2450E-01	6.1931E-01	6.1363E-01	6.0735E-01
5.5000E+01	7.3200E-01	7.3000E-01	7.2907E-01	7.2954E-01
6.0000E+01	8.1439E-01	8.1462E-01	8.1681E-01	8.2145E-01
6.5000E+01	8.7440E-01	8.7589E-01	8.7961E-01	8.8602E-01
7.0000E+01	9.1650E-01	9.1850E-01	9.2260E-01	9.2911E-01
7.5000E+01	9.4522E-01	9.4726E-01	9.5107E-01	9.5679E-01
8.0000E+01	9.6441E-01	9.6624E-01	9.6946E-01	9.7406E-01
8.5000E+01	9.7704E-01	9.7856E-01	9.8111E-01	9.8461E-01
9.0000E+01	9.8526E-01	9.8646E-01	9.8839E-01	9.9094E-01
9.5000E+01	9.9057E-01	9.9148E-01	9.9290E-01	9.9470E-01
1.0000E+02	9.9397E-01	9.9464E-01	9.9567E-01	9.9691E-01

Gamma

Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	7.2334E-12	1.8390E-12	2.0276E-13	6.9267E-15
1.0000E+01	1.4139E-06	7.2829E-07	2.5205E-07	4.9786E-08
1.5000E+01	1.8932E-04	1.3153E-04	7.3756E-05	3.0494E-05
2.0000E+01	2.2528E-03	1.8392E-03	1.3360E-03	8.2017E-04
2.5000E+01	8.6933E-03	7.8126E-03	6.6135E-03	5.1251E-03
3.0000E+01	1.8099E-02	1.7264E-02	1.6056E-02	1.4350E-02
3.5000E+01	2.5953E-02	2.5691E-02	2.5315E-02	2.4683E-02
4.0000E+01	2.9260E-02	2.9618E-02	3.0194E-02	3.0968E-02
4.5000E+01	2.8046E-02	2.8736E-02	2.9814E-02	3.1368E-02
5.0000E+01	2.4005E-02	2.4715E-02	2.5793E-02	2.7346E-02
5.5000E+01	1.8950E-02	1.9500E-02	2.0296E-02	2.1401E-02
6.0000E+01	1.4105E-02	1.4449E-02	1.4903E-02	1.5476E-02
6.5000E+01	1.0054E-02	1.0221E-02	1.0396E-02	1.0555E-02
7.0000E+01	6.9401E-03	6.9861E-03	6.9806E-03	6.8890E-03

7.5000E+01		4.6779E-03	4.6538E-03	4.5552E-03	4.3505E-03
8.0000E+01		3.0978E-03	3.0414E-03	2.9096E-03	2.6800E-03
8.5000E+01		2.0250E-03	1.9598E-03	1.8291E-03	1.6205E-03
9.0000E+01		1.3114E-03	1.2499E-03	1.1365E-03	9.6637E-04
9.5000E+01		8.4362E-04	7.9138E-04	7.0024E-04	5.7052E-04
1.0000E+02		5.4033E-04	4.9856E-04	4.2893E-04	3.3442E-04

Lambda

-----

Stock Price		Time to Maturity (months)			
		5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
-----					
0.0000E+00		0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00		3.6558E-11	8.6441E-12	8.6672E-13	2.6259E-14
1.0000E+01		2.8583E-05	1.3693E-05	4.3098E-06	7.5495E-07
1.5000E+01		8.6115E-03	5.5645E-03	2.8375E-03	1.0404E-03
2.0000E+01		1.8217E-01	1.3832E-01	9.1376E-02	4.9748E-02
2.5000E+01		1.0984E+00	9.1808E-01	7.0676E-01	4.8574E-01
3.0000E+01		3.2931E+00	2.9214E+00	2.4708E+00	1.9584E+00
3.5000E+01		6.4272E+00	5.9173E+00	5.3025E+00	4.5851E+00
4.0000E+01		9.4643E+00	8.9101E+00	8.2604E+00	7.5135E+00
4.5000E+01		1.1481E+01	1.0941E+01	1.0323E+01	9.6323E+00
5.0000E+01		1.2132E+01	1.1617E+01	1.1026E+01	1.0367E+01
5.5000E+01		1.1588E+01	1.1091E+01	1.0498E+01	9.8169E+00
6.0000E+01		1.0265E+01	9.7801E+00	9.1734E+00	8.4486E+00
6.5000E+01		8.5872E+00	8.1198E+00	7.5104E+00	6.7621E+00
7.0000E+01		6.8747E+00	6.4363E+00	5.8487E+00	5.1188E+00
7.5000E+01		5.3194E+00	4.9219E+00	4.3812E+00	3.7109E+00
8.0000E+01		4.0081E+00	3.6599E+00	3.1840E+00	2.6009E+00
8.5000E+01		2.9578E+00	2.6623E+00	2.2597E+00	1.7754E+00
9.0000E+01		2.1474E+00	1.9036E+00	1.5741E+00	1.1870E+00
9.5000E+01		1.5392E+00	1.3429E+00	1.0806E+00	7.8078E-01
1.0000E+02		1.0923E+00	9.3740E-01	7.3341E-01	5.0711E-01

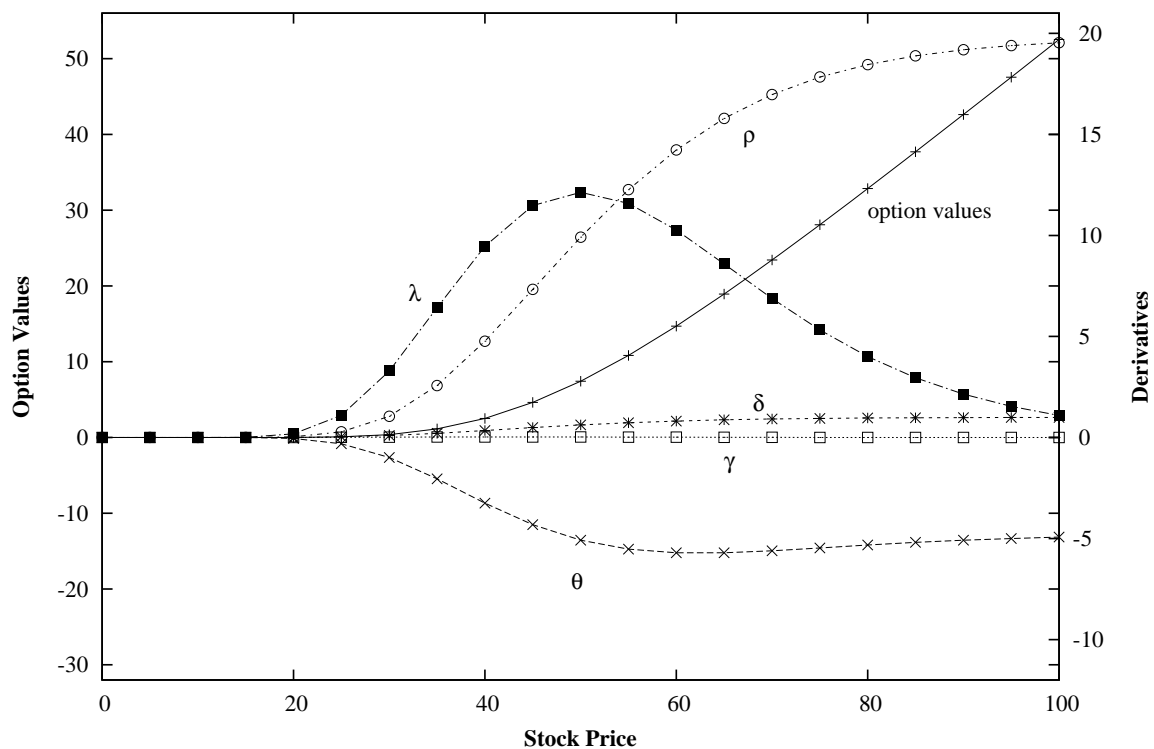
Rho

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Stock Price		Time to Maturity (months)			
		5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
-----					
0.0000E+00		0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00		3.2110E-12	6.9908E-13	6.3513E-14	1.7073E-15
1.0000E+01		3.5302E-06	1.5579E-06	4.4470E-07	6.9214E-08
1.5000E+01		1.3876E-03	8.2648E-04	3.8273E-04	1.2492E-04
2.0000E+01		3.7221E-02	2.6077E-02	1.5671E-02	7.6142E-03
2.5000E+01		2.8124E-01	2.1719E-01	1.5247E-01	9.3836E-02
3.0000E+01		1.0531E+00	8.6478E-01	6.6907E-01	4.7709E-01
3.5000E+01		2.5718E+00	2.1971E+00	1.8086E+00	1.4156E+00
4.0000E+01		4.7641E+00	4.1750E+00	3.5750E+00	2.9673E+00
4.5000E+01		7.3281E+00	6.5270E+00	5.7279E+00	4.9280E+00
5.0000E+01		9.9152E+00	8.9196E+00	7.9427E+00	6.9774E+00
5.5000E+01		1.2262E+01	1.1095E+01	9.9592E+00	8.8448E+00
6.0000E+01		1.4232E+01	1.2915E+01	1.1637E+01	1.0383E+01
6.5000E+01		1.5791E+01	1.4348E+01	1.2942E+01	1.1557E+01
7.0000E+01		1.6973E+01	1.5424E+01	1.3907E+01	1.2403E+01
7.5000E+01		1.7838E+01	1.6204E+01	1.4594E+01	1.2987E+01
8.0000E+01		1.8457E+01	1.6755E+01	1.5067E+01	1.3376E+01
8.5000E+01		1.8890E+01	1.7135E+01	1.5387E+01	1.3629E+01
9.0000E+01		1.9189E+01	1.7393E+01	1.5599E+01	1.3790E+01
9.5000E+01		1.9393E+01	1.7567E+01	1.5738E+01	1.3891E+01
1.0000E+02		1.9531E+01	1.7683E+01	1.5827E+01	1.3954E+01



**Example Program 1**  
Option Values and Derivatives at 5 Months to Maturity



**Example Program 2**  
Option Values and Derivatives at 3.5 Months to Maturity

