

NAG Library Function Document

nag_bessel_k_alpha (s18egc)

1 Purpose

nag_bessel_k_alpha (s18egc) returns a sequence of values for the modified Bessel functions $K_{\alpha+n}(x)$ for real $x > 0$, selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$.

2 Specification

```
#include <nag.h>
#include <nags.h>

void nag_bessel_k_alpha (double x, Integer ia, Integer ja, Integer nl,
                        double b[], NagError *fail)
```

3 Description

nag_bessel_k_alpha (s18egc) evaluates a sequence of values for the modified Bessel function of the second kind $K_{\alpha}(x)$, where x is real and non-negative and $\alpha \in \{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}\}$ is the order. The $(N+1)$ -member sequence is generated for orders $\alpha, \alpha+1, \dots, \alpha+N$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: **x** > 0.0.

2: **ia** – Integer *Input*

3: **ja** – Integer *Input*

On entry: the numerator i and denominator j , respectively, of the order $\alpha = i/j$ of the first member in the required sequence of function values. Only the following combinations of pairs of values of i and j are allowed:

$i = 0$ and $j = 1$ corresponds to $\alpha = 0$;

$i = 1$ and $j = 2$ corresponds to $\alpha = \frac{1}{2}$;

$i = 1$ and $j = 3$ corresponds to $\alpha = \frac{1}{3}$;

$i = 1$ and $j = 4$ corresponds to $\alpha = \frac{1}{4}$;

$i = 2$ and $j = 3$ corresponds to $\alpha = \frac{2}{3}$;

$i = 3$ and $j = 4$ corresponds to $\alpha = \frac{3}{4}$.

Constraint: **ia** and **ja** must constitute a valid pair $(\mathbf{ia}, \mathbf{ja}) = (0, 1), (1, 2), (1, 3), (1, 4), (2, 3)$ or $(3, 4)$.

- 4: **nl** – Integer *Input*
On entry: the value of N . Note that the order of the last member in the required sequence of function values is given by $\alpha + N$.
Constraint: $0 \leq \mathbf{nl} \leq 100$.
- 5: **b[nl + 1]** – double *Output*
On exit: with **fail.code** = NE_NOERROR or **fail.code** = NW_SOME_PRECISION_LOSS, the required sequence of function values: **b**(n) contains $K_{\alpha+n}(x)$ for $n = 0, 1, \dots, N$.
- 6: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_INT

On entry, **nl** = $\langle value \rangle$.
Constraint: $0 \leq \mathbf{nl} \leq 100$.

NE_INT_2

On entry, **ia** = $\langle value \rangle$, **ja** = $\langle value \rangle$.
Constraint: **ia** and **ja** must constitute a valid pair (**ia,ja**).

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_OVERFLOW_LIKELY

The evaluation has been abandoned due to the likelihood of overflow.

NE_REAL

On entry, **x** = $\langle value \rangle$.
Constraint: **x** > 0.0.

NE_TERMINATION_FAILURE

The evaluation has been abandoned due to failure to satisfy the termination condition.

NE_TOTAL_PRECISION_LOSS

The evaluation has been abandoned due to total loss of precision.

NW_SOME_PRECISION_LOSS

The evaluation has been completed but some precision has been lost.

7 Accuracy

All constants in the underlying function are specified to approximately 18 digits of precision. If t denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$. Because of errors in argument reduction when computing elementary functions inside the underlying function, the actual number of correct digits is limited, in general, by $p - s$, where $s \approx \max(1, |\log_{10} x|)$ represents

the number of digits lost due to the argument reduction. Thus the larger the value of x , the less the precision in the result.

8 Parallelism and Performance

nag_bessel_k_alpha (s18egc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

The example program evaluates $K_0(x)$, $K_1(x)$, $K_2(x)$ and $K_3(x)$ at $x = 0.5$, and prints the results.

10.1 Program Text

```
/* nag_bessel_k_alpha (s18egc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0, i, ia, ja, nl;
    NagError fail;
    double alpha, *b = 0, x;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[^\\n]");
#else
    scanf("%*[^\\n]");
#endif
    printf("nag_bessel_k_alpha (s18egc) Example Program Results\\n");
    if (!(b = NAG_ALLOC(101, double)))
    {
        printf("Allocation failure\\n");
        exit_status = -1;
        goto END;
    }

#ifdef _WIN32
    while (scanf_s
        ("%lf %" NAG_IFMT " %" NAG_IFMT " %" NAG_IFMT "%*[^\\n]", &x, &ia,
         &ja, &nl) != EOF) {
#else
    while (scanf
        ("%lf %" NAG_IFMT " %" NAG_IFMT " %" NAG_IFMT "%*[^\\n]", &x, &ia,
         &ja, &nl) != EOF) {
#endif
        printf(" x      ia      ja      nl\\n");
        printf("%4.1f %6" NAG_IFMT " %6" NAG_IFMT " %6" NAG_IFMT "\\n\\n", x, ia,
```

```

        ja, nl);
/* nag_bessel_k_alpha (s18egc).
 * Modified Bessel functions K_(alpha+n)(x) for real
 * x > 0, selected values of alpha >= 0 and
 * n = 0,1,...,N
 */
nag_bessel_k_alpha(x, ia, ja, nl, b, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_bessel_k_alpha (s18egc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
printf(" Requested values of K_alpha(x)\n\n");
alpha = (double) ia / (double) ja;
printf("      alpha      K_alpha(x)\n");
for (i = 0; i <= nl; ++i) {
    printf(" %13.4e %13.4e\n", alpha, b[i]);
    alpha += 1.0;
}
}
END:
    NAG_FREE(b);
    return exit_status;
}

```

10.2 Program Data

nag_bessel_k_alpha (s18egc) Example Program Data
 0.5 0 1 3 : Values of x, ia, ja and nl

10.3 Program Results

nag_bessel_k_alpha (s18egc) Example Program Results

x	ia	ja	nl
0.5	0	1	3

Requested values of K_alpha(x)

alpha	K_alpha(x)
0.0000e+00	9.2442e-01
1.0000e+00	1.6564e+00
2.0000e+00	7.5502e+00
3.0000e+00	6.2058e+01
