

NAG Library Function Document

nag_erf (s15aec)

1 Purpose

nag_erf (s15aec) returns the value of the error function $\operatorname{erf}(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_erf (double x)
```

3 Description

nag_erf (s15aec) calculates an approximate value for the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = 1 - \operatorname{erfc}(x).$$

Let \hat{x} be the root of the equation $\operatorname{erfc}(x) - \operatorname{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\operatorname{erf}(x)$ is based on the following rational Chebyshev expansion for $\operatorname{erf}(x)$:

$$\operatorname{erf}(x) \approx x R_{\ell,m}(x^2),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\operatorname{erf}(x)$ is based on a rational Chebyshev expansion for $\operatorname{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\operatorname{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\operatorname{erfc}(x) \approx \frac{e^{x^2}}{x} \left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d} , where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_decimal_digits (X02BEC)).

For $|x| \geq x_{\text{hi}}$ there is a danger of setting underflow in $\operatorname{erfc}(x)$ (the value of x_{hi} is given in the Users' Note for your implementation). For $x \geq x_{\text{hi}}$, nag_erf (s15aec) returns $\operatorname{erf}(x) = 1$; for $x \leq -x_{\text{hi}}$ it returns $\operatorname{erf}(x) = -1$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J (1969) Rational Chebyshev approximations for the error function *Math.Comp.* **23** 631–637

5 Arguments

1: **x** – double

Input

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

See Section 7 in nag_erfc (s15adc).

8 Parallelism and Performance

nag_erf (s15aec) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_erf (s15aec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[^\\n]");
#else
    scanf("%*[^\\n]");
#endif
    printf("nag_erf (s15aec) Example Program Results\\n");
    printf("      x              y\\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
#else
    while (scanf("%lf", &x) != EOF)
#endif
    {
        /* nag_erf (s15aec).
         * Error function erf(x)
         */
        y = nag_erf(x);
        printf("%12.3e%12.3e\\n", x, y);
    }

    return exit_status;
}
```

10.2 Program Data

```
nag_erf (s15aec) Example Program Data
      -6.0
      -4.5
      -1.0
       1.0
       4.5
       6.0
```

10.3 Program Results

```
nag_erf (s15aec) Example Program Results
      x           y
-6.000e+00  -1.000e+00
-4.500e+00  -1.000e-00
-1.000e+00  -8.427e-01
 1.000e+00   8.427e-01
 4.500e+00   1.000e-00
 6.000e+00   1.000e+00
```
