

# NAG Library Function Document

## nag\_erfc (s15adc)

### 1 Purpose

nag\_erfc (s15adc) returns the value of the complementary error function,  $\text{erfc}(x)$ .

### 2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_erfc (double x)
```

### 3 Description

nag\_erfc (s15adc) calculates an approximate value for the complement of the error function

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt = 1 - \text{erf}(x).$$

Let  $\hat{x}$  be the root of the equation  $\text{erfc}(x) - \text{erf}(x) = 0$  (then  $\hat{x} \approx 0.46875$ ). For  $|x| \leq \hat{x}$  the value of  $\text{erfc}(x)$  is based on the following rational Chebyshev expansion for  $\text{erf}(x)$ :

$$\text{erf}(x) \approx x R_{\ell,m}(x^2),$$

where  $R_{\ell,m}$  denotes a rational function of degree  $\ell$  in the numerator and  $m$  in the denominator.

For  $|x| > \hat{x}$  the value of  $\text{erfc}(x)$  is based on a rational Chebyshev expansion for  $\text{erfc}(x)$ : for  $\hat{x} < |x| \leq 4$  the value is based on the expansion

$$\text{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for  $|x| > 4$  it is based on the expansion

$$\text{erfc}(x) \approx \frac{e^{x^2}}{x} \left( \frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).$$

For each expansion, the specific values of  $\ell$  and  $m$  are selected to be minimal such that the maximum relative error in the expansion is of the order  $10^{-d}$ , where  $d$  is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag\_decimal\_digits (X02BEC)).

For  $|x| \geq x_{\text{hi}}$  there is a danger of setting underflow in  $\text{erfc}(x)$  (the value of  $x_{\text{hi}}$  is given in the Users' Note for your implementation). For  $x \geq x_{\text{hi}}$ , nag\_erfc (s15adc) returns  $\text{erfc}(x) = 0$ ; for  $x \leq -x_{\text{hi}}$  it returns  $\text{erfc}(x) = 2$ .

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J (1969) Rational Chebyshev approximations for the error function *Math.Comp.* **23** 631–637

### 5 Arguments

1: **x** – double

*Input*

*On entry:* the argument  $x$  of the function.

## 6 Error Indicators and Warnings

None.

## 7 Accuracy

If  $\delta$  and  $\epsilon$  are relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{2xe^{-x^2}}{\sqrt{\pi} \operatorname{erfc}(x)} \delta \right|.$$

That is, the relative error in the argument,  $x$ , is amplified by a factor  $\frac{2xe^{-x^2}}{\sqrt{\pi} \operatorname{erfc}(x)}$  in the result.

The behaviour of this factor is shown in Figure 1.

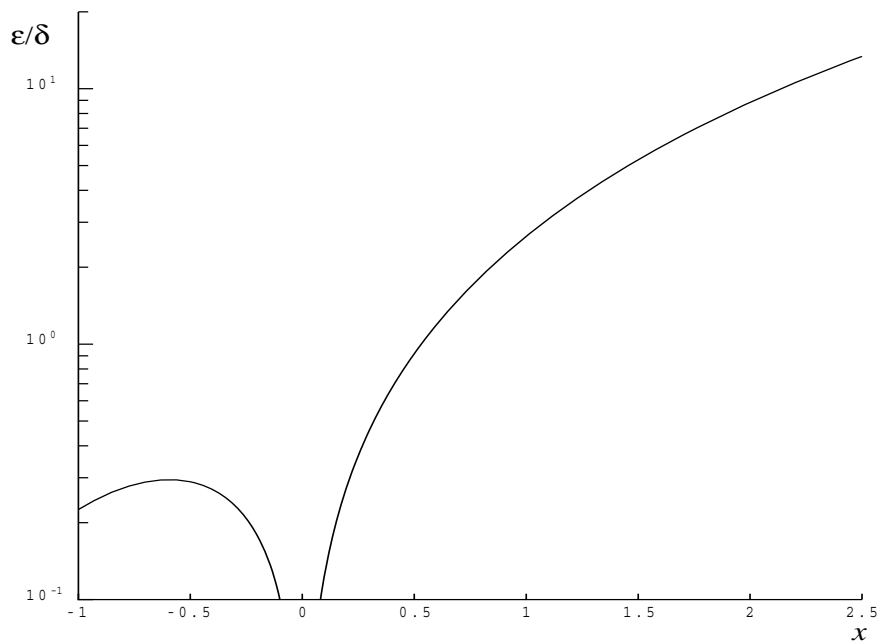


Figure 1

It should be noted that near  $x = 0$  this factor behaves as  $\frac{2x}{\sqrt{\pi}}$  and hence the accuracy is largely determined by the *machine precision*. Also for large negative  $x$ , where the factor is  $\sim \frac{xe^{-x^2}}{\sqrt{\pi}}$ , accuracy is mainly limited by *machine precision*. However, for large positive  $x$ , the factor becomes  $\sim 2x^2$  and to an extent relative accuracy is necessarily lost. The absolute accuracy  $E$  is given by

$$E \simeq \frac{2xe^{-x^2}}{\sqrt{\pi}} \delta$$

so absolute accuracy is guaranteed for all  $x$ .

## 8 Parallelism and Performance

nag\_erfc (s15adc) is not threaded in any implementation.

## 9 Further Comments

None.

## 10 Example

This example reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

### 10.1 Program Text

```
/* nag_erfc (s15adc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_erfc (s15adc) Example Program Results\n");
    printf("      x              y\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
#else
    while (scanf("%lf", &x) != EOF)
#endif
    {
        /* nag_erfc (s15adc).
         * Complement of error function erfc(x)
         */
        y = nag_erfc(x);
        printf("%12.3e%12.3e\n", x, y);
    }

    return exit_status;
}
```

### 10.2 Program Data

```
nag_erfc (s15adc) Example Program Data
      -10.0
       -1.0
        0.0
        1.0
       10.0
```

### 10.3 Program Results

nag\_erfc (s15adc) Example Program Results

x	y
-1.000e+01	2.000e+00
-1.000e+00	1.843e+00
0.000e+00	1.000e-00
1.000e+00	1.573e-01
1.000e+01	2.088e-45

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