

NAG Library Function Document

nag_cumul_normal (s15abc)

1 Purpose

nag_cumul_normal (s15abc) returns the value of the cumulative Normal distribution function, $P(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>

double nag_cumul_normal (double x)
```

3 Description

nag_cumul_normal (s15abc) evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du.$$

The function is based on the fact that

$$P(x) = \frac{1}{2} \operatorname{erfc} \left(\frac{-x}{\sqrt{2}} \right)$$

and it calls nag_erfc (s15adc) to obtain a value of *erfc* for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*
On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

Because of its close relationship with *erfc*, the accuracy of this function is very similar to that in nag_erfc (s15adc). If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)} \delta \right|$$

so that the relative error in the argument, x , is amplified by a factor, $\frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by *machine precision*.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E , is given by

$$|E| \simeq \left| \frac{x e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \delta \right|$$

so absolute accuracy can be guaranteed for all x .

8 Parallelism and Performance

nag_cumul_normal (s15abc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_cumul_normal (s15abc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_cumul_normal (s15abc) Example Program Results\n");
    printf("      x              y\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
    #else
    while (scanf("%lf", &x) != EOF)
    #endif
    {
        /* nag_cumul_normal (s15abc).
         * Cumulative Normal distribution function P(x)
         */
        y = nag_cumul_normal(x);
    }
}
```

```
    printf("%12.3e%12.3e\n", x, y);  
}  
  
    return exit_status;  
}
```

10.2 Program Data

```
nag_cumul_normal (s15abc) Example Program Data  
      -20.0  
      -1.0  
       0.0  
       1.0  
       2.0  
      20.0
```

10.3 Program Results

```
nag_cumul_normal (s15abc) Example Program Results  
      x           y  
-2.000e+01  2.754e-89  
-1.000e+00  1.587e-01  
 0.000e+00  5.000e-01  
 1.000e+00  8.413e-01  
 2.000e+00  9.772e-01  
 2.000e+01  1.000e+00
```
