

NAG Library Function Document

nag_incomplete_beta (s14ccc)

1 Purpose

nag_incomplete_beta (s14ccc) computes values for the incomplete beta function $I_x(a, b)$ and its complement $1 - I_x(a, b)$.

2 Specification

```
#include <nag.h>
#include <nags.h>

void nag_incomplete_beta (double a, double b, double x, double *w,
                          double *wl, NagError *fail)
```

3 Description

nag_incomplete_beta (s14ccc) evaluates the incomplete beta function and its complement in the normalized form

$$I_x(a, b) = \frac{1}{B(a, b)} \int_0^x t^{a-1} (1-t)^{b-1} dt$$

$$1 - I_x(a, b) = I_y(b, a), \text{ where } y = 1 - x,$$

with

$$0 \leq x \leq 1,$$

$$a \geq 0 \text{ and } b \geq 0,$$

and the beta function $B(a, b)$ is defined as $B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ where $\Gamma(y)$ is the gamma function.

Several methods are used to evaluate the functions depending on the arguments a , b and x . The methods include Wise's asymptotic expansion (see Wise (1950)) when $a > b$, continued fraction derived by DiDonato and Morris (1992) when $a, b > 1$, and power series when $b \leq 1$ or $b \times x \leq 0.7$. When both a and b are large, specifically $a, b \geq 15$, the DiDonato and Morris (1992) asymptotic expansion is employed for greater efficiency.

Once either $I_x(a, b)$ or $I_y(b, a)$ is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of $I_x(a, b)$ and $I_y(b, a)$ is computed first.

nag_incomplete_beta (s14ccc) is derived from BRATIO in DiDonato and Morris (1992).

4 References

DiDonato A R and Morris A H (1992) Algorithm 708: Significant digit computation of the incomplete beta function ratios *ACM Trans. Math. Software* **18** 360–373

Wise M E (1950) The incomplete beta function as a contour integral and a quickly converging series for its inverse *Biometrika* **37** 208–218

5 Arguments

- 1: **a** – double *Input*
On entry: the argument a of the function.
Constraint: $\mathbf{a} \geq 0.0$.

- 2: **b** – double *Input*
On entry: the argument b of the function.
Constraints:
 $\mathbf{b} \geq 0.0$;
either $\mathbf{b} \neq 0.0$ or $\mathbf{a} \neq 0.0$.

- 3: **x** – double *Input*
On entry: x , upper limit of integration.
Constraints:
 $0.0 \leq \mathbf{x} \leq 1.0$;
either $\mathbf{x} \neq 0.0$ or $\mathbf{a} \neq 0.0$;
either $1 - \mathbf{x} \neq 0.0$ or $\mathbf{b} \neq 0.0$.

- 4: **w** – double * *Output*
On exit: the value of the incomplete beta function $I_x(a, b)$ evaluated from zero to x .

- 5: **w1** – double * *Output*
On exit: the value of the complement of the incomplete beta function $1 - I_x(a, b)$, i.e., the incomplete beta function evaluated from x to one.

- 6: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

On entry, **a** = $\langle value \rangle$.

Constraint: **a** \geq 0.0.

On entry, **b** = $\langle value \rangle$.

Constraint: **b** \geq 0.0.

On entry, **x** = $\langle value \rangle$.

Constraint: $0.0 \leq \mathbf{x} \leq 1.0$.

NE_REAL_2

On entry, $1.0 - \mathbf{x}$ and **b** were zero.

Constraint: $1.0 - \mathbf{x}$ or **b** must be nonzero.

On entry, **a** and **b** were zero.

Constraint: **a** or **b** must be nonzero.

On entry, **x** and **a** were zero.

Constraint: **x** or **a** must be nonzero.

7 Accuracy

nag_incomplete_beta (s14ccc) is designed to maintain relative accuracy for all arguments. For very tiny results (of the order of *machine precision* or less) some relative accuracy may be lost – loss of three or four decimal places has been observed in experiments. For other arguments full relative accuracy may be expected.

8 Parallelism and Performance

nag_incomplete_beta (s14ccc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the arguments *a* and *b* from a file, evaluates the function and its complement for 10 different values of *x* and prints the results.

10.1 Program Text

```
/* nag_incomplete_beta (s14ccc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double a, b, x, w, wl;
    NagError fail;
```

```

INIT_FAIL(fail);

printf("nag_incomplete_beta (s14ccc) Example Program Results\n");

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    printf("%4s%7s%7s%13s%13s\n", "a", "b", "x", "Ix(a,b)", "1-Ix(a,b)");
#ifdef _WIN32
    while (scanf_s("%lf %lf %lf", &a, &b, &x) != EOF)
#else
    while (scanf("%lf %lf %lf", &a, &b, &x) != EOF)
#endif
    {
        /* nag_incomplete_beta (s14ccc).
         * Incomplete beta function Ix(a,b) and its complement 1-Ix(a,b)
         */
        nag_incomplete_beta(a, b, x, &w, &wl, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_incomplete_beta (s14ccc).\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%6.2f%7.2f%7.2f%12.4e%12.4e\n", a, b, x, w, wl);
    }

END:
    return exit_status;
}

```

10.2 Program Data

```

nag_incomplete_beta (s14ccc) Example Program Data
5.3  10.1  0.01
1.0  0.2   0.02
0.2  1.0   0.04
1.2  5.1   0.05
7.0  2.5   0.08
1.0  1.0   0.09
10.0 1.5   0.10

```

10.3 Program Results

```

nag_incomplete_beta (s14ccc) Example Program Results
a      b      x      Ix(a,b)      1-Ix(a,b)
5.30  10.10  0.01  6.4755e-08  1.0000e+00
1.00   0.20  0.02  4.0324e-03  9.9597e-01
0.20   1.00  0.04  5.2531e-01  4.7469e-01
1.20   5.10  0.05  1.6101e-01  8.3899e-01
7.00   2.50  0.08  3.3490e-07  1.0000e+00
1.00   1.00  0.09  9.0000e-02  9.1000e-01
10.00  1.50  0.10  3.5279e-10  1.0000e+00

```
