

# NAG Library Function Document

## nag\_tanh (s10aac)

### 1 Purpose

nag\_tanh (s10aac) returns a value for the hyperbolic tangent,  $\tanh x$ .

### 2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_tanh (double x)
```

### 3 Description

nag\_tanh (s10aac) calculates an approximate value for the hyperbolic tangent of its argument,  $\tanh x$ .

For  $|x| \leq 1$  it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x \sum_{r=0} a_r T_r(t)$$

where  $-1 \leq x \leq 1$ ,  $-1 \leq t \leq 1$ , and  $t = 2x^2 - 1$ .

For  $1 < |x| < E_1$  (see the Users' Note for your implementation for value of  $E_1$ )

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For  $|x| \geq E_1$ ,  $\tanh x = \text{sign } x$  to within the representation accuracy of the machine and so this approximation is used.

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

### 5 Arguments

1: **x** – double

*Input*

*On entry:* the argument  $x$  of the function.

### 6 Error Indicators and Warnings

None.

### 7 Accuracy

If  $\delta$  and  $\epsilon$  are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{2x}{\sinh 2x} \delta \right|.$$

That is, a relative error in the argument,  $x$ , is amplified by a factor approximately  $\frac{2x}{\sinh 2x}$ , in the result.

The equality should hold if  $\delta$  is greater than the *machine precision* ( $\delta$  due to data errors etc.) but if  $\delta$  is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

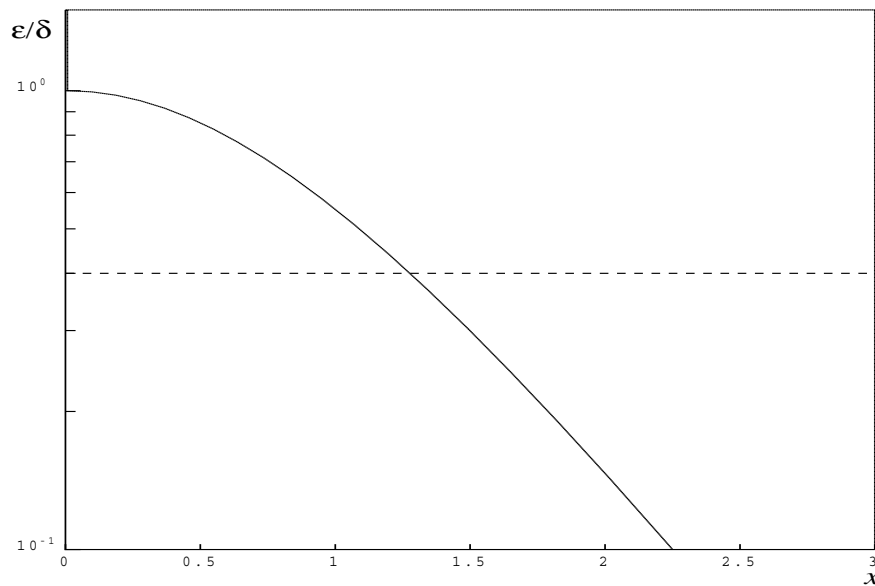


Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from  $x = 0$  the accuracy will eventually be limited entirely by the precision of machine representation.

## 8 Parallelism and Performance

nag\_tanh (s10aac) is not threaded in any implementation.

## 9 Further Comments

None.

## 10 Example

This example reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

### 10.1 Program Text

```
/* nag_tanh (s10aac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
```

```

double x, y;

/* Skip heading in data file */
#ifdef _WIN32
scanf_s("%*[\n]");
#else
scanf("%*[\n]");
#endif
printf("nag_tanh (s10aac) Example Program Results\n");
printf("      x      y\n");
#ifdef _WIN32
while (scanf_s("%lf", &x) != EOF)
#else
while (scanf("%lf", &x) != EOF)
#endif
{
    /* nag_tanh (s10aac).
     * Hyperbolic tangent, tanh x
     */
    y = nag_tanh(x);
    printf("%12.1f%12.5f\n", x, y);
}

return exit_status;
}

```

## 10.2 Program Data

```

nag_tanh (s10aac) Example Program Data
      -20.0
      -5.0
       0.5
       5.0

```

## 10.3 Program Results

```

nag_tanh (s10aac) Example Program Results
      x      y
    -20.0    -1.00000
     -5.0    -0.99991
       0.5     0.46212
       5.0     0.99991

```

---