

# NAG Library Function Document

## nag\_tsa\_arma\_roots (g13dxc)

### 1 Purpose

nag\_tsa\_arma\_roots (g13dxc) calculates the zeros of a vector autoregressive (or moving average) operator.

### 2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_arma_roots (Integer k, Integer ip, const double par[],
                        double rr[], double ri[], double rmod[], NagError *fail)
```

### 3 Description

Consider the vector autoregressive moving average (VARMA) model

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p(W_{t-p} - \mu) + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}, \quad (1)$$

where  $W_t$  denotes a vector of  $k$  time series and  $\epsilon_t$  is a vector of  $k$  residual series having zero mean and a constant variance-covariance matrix. The components of  $\epsilon_t$  are also assumed to be uncorrelated at non-simultaneous lags.  $\phi_1, \phi_2, \dots, \phi_p$  denotes a sequence of  $k$  by  $k$  matrices of autoregressive (AR) parameters and  $\theta_1, \theta_2, \dots, \theta_q$  denotes a sequence of  $k$  by  $k$  matrices of moving average (MA) parameters.  $\mu$  is a vector of length  $k$  containing the series means. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk}$$

where  $I$  denotes the  $k$  by  $k$  identity matrix.

The model (1) is said to be stationary if the eigenvalues of  $A(\phi)$  lie inside the unit circle. Similarly let

$$B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}$$

Then the model is said to be invertible if the eigenvalues of  $B(\theta)$  lie inside the unit circle.

nag\_tsa\_arma\_roots (g13dxc) returns the  $pk$  eigenvalues of  $A(\phi)$  (or the  $qk$  eigenvalues of  $B(\theta)$ ) along with their moduli, in descending order of magnitude. Thus to check for stationarity or invertibility you should check whether the modulus of the largest eigenvalue is less than one.

### 4 References

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

## 5 Arguments

- 1:    **k** – Integer *Input*  
*On entry:*  $k$ , the dimension of the multivariate time series.  
*Constraint:*  $k \geq 1$ .
  
- 2:    **ip** – Integer *Input*  
*On entry:* the number of AR (or MA) parameter matrices,  $p$  (or  $q$ ).  
*Constraint:*  $ip \geq 1$ .
  
- 3:    **par**[**ip** × **k** × **k**] – const double *Input*  
*On entry:* the AR (or MA) parameter matrices read in row by row in the order  $\phi_1, \phi_2, \dots, \phi_p$  (or  $\theta_1, \theta_2, \dots, \theta_q$ ). That is, **par**[( $l-1$ ) ×  $k \times k + (i-1) \times k + j-1$ ] must be set equal to the ( $i, j$ )th element of  $\phi_l$ , for  $l = 1, 2, \dots, p$  (or the ( $i, j$ )th element of  $\theta_l$ , for  $l = 1, 2, \dots, q$ ).
  
- 4:    **rr**[**k** × **ip**] – double *Output*  
*On exit:* the real parts of the eigenvalues.
  
- 5:    **ri**[**k** × **ip**] – double *Output*  
*On exit:* the imaginary parts of the eigenvalues.
  
- 6:    **rmod**[**k** × **ip**] – double *Output*  
*On exit:* the moduli of the eigenvalues.
  
- 7:    **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_EIGENVALUES

An excessive number of iterations have been required to calculate the eigenvalues.

### NE\_INT

On entry, **ip** =  $\langle value \rangle$ .

Constraint:  $ip \geq 1$ .

On entry, **k** =  $\langle value \rangle$ .

Constraint:  $k \geq 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

## NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The accuracy of the results depends on the original matrix and the multiplicity of the roots.

## 8 Parallelism and Performance

nag\_tsa\_arma\_roots (g13dxc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_tsa\_arma\_roots (g13dxc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The time taken is approximately proportional to  $kp^3$  (or  $kq^3$ ).

## 10 Example

This example finds the eigenvalues of  $A(\phi)$  where  $k = 2$  and  $p = 1$  and  $\phi_1 = \begin{bmatrix} 0.802 & 0.065 \\ 0.000 & 0.575 \end{bmatrix}$ .

### 10.1 Program Text

```
/* nag_tsa_arma_roots (g13dxc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    Integer exit_status, i, ip, k, npar;
    NagError fail;

    /* Arrays */
    double *par = 0, *ri = 0, *rmod = 0, *rr = 0;

    INIT_FAIL(fail);

    exit_status = 0;

    printf("nag_tsa_arma_roots (g13dxc) Example Program Results\n");
```

```

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &k, &ip);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &k, &ip);
#endif

if (k > 0 && ip > 0) {
    /* Allocate arrays */
    if (!(par = NAG_ALLOC(k * k * ip, double)) ||
        !(ri = NAG_ALLOC(k * ip, double)) ||
        !(rmod = NAG_ALLOC(k * ip, double)) ||
        !(rr = NAG_ALLOC(k * ip, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read the AR (or MA) parameters */
    npar = ip * k * k;
    for (i = 1; i <= npar; ++i)
#ifdef _WIN32
        scanf_s("%lf", &par[i - 1]);
#else
        scanf("%lf", &par[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* nag_tsa_arma_roots (g13dxc).
     * Calculates the zeros of a vector autoregressive (or
     * moving average) operator
     */
    nag_tsa_arma_roots(k, ip, par, rr, ri, rmod, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_arma_roots (g13dxc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
    printf("\n");
    printf("          Eigenvalues          Moduli\n");
    printf("          -----\n");
    for (i = 1; i <= k * ip; ++i) {
        if (ri[i - 1] >= 0.0)
            printf("%10.3f + %6.3f i %8.3f\n", rr[i - 1], ri[i - 1],
                rmod[i - 1]);
        else
            printf("%10.3f - %6.3f i %8.3f\n", rr[i - 1], -ri[i - 1],
                rmod[i - 1]);
    }
}
else
    printf(" Either k or ip is out of range\n");

END:
    NAG_FREE(par);
    NAG_FREE(ri);

```

```

    NAG_FREE(rmod);
    NAG_FREE(rr);

    return exit_status;
}

```

## 10.2 Program Data

```

nag_tsa_arma_roots (g13dxc) Example Program Data
  2 1
  0.802 0.065
  0.000 0.575

```

## 10.3 Program Results

```

nag_tsa_arma_roots (g13dxc) Example Program Results

```

Eigenvalues				Moduli
-----				-----
0.802	+	0.000	i	0.802
0.575	+	0.000	i	0.575

---