

## NAG Library Function Document

### nag\_tsa\_cross\_spectrum\_bivar (g13cec)

#### 1 Purpose

For a bivariate time series, nag\_tsa\_cross\_spectrum\_bivar (g13cec) calculates the cross amplitude spectrum and squared coherency, together with lower and upper bounds from the univariate and bivariate (cross) spectra.

#### 2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_cross_spectrum_bivar (const double xg[], const double yg[],
    const Complex xyg[], Integer ng, const double stats[], double ca[],
    double calw[], double caup[], double *t, double sc[], double sclw[],
    double scup[], NagError *fail)
```

#### 3 Description

Estimates of the cross amplitude spectrum  $A(\omega)$  and squared coherency  $W(\omega)$  are calculated for each frequency  $\omega$  as

$$A(\omega) = |f_{xy}(\omega)| = \sqrt{cf(\omega)^2 + qf(\omega)^2} \text{ and}$$

$$W(\omega) = \frac{|f_{xy}(\omega)|^2}{f_{xx}(\omega)f_{yy}(\omega)}$$

where:

$cf(\omega)$  and  $qf(\omega)$  are the co-spectrum and quadrature spectrum estimates between the series, i.e., the real and imaginary parts of the cross spectrum  $f_{xy}(\omega)$  as obtained using nag\_tsa\_spectrum\_bivar\_cov (g13ccc) or nag\_tsa\_spectrum\_bivar (g13cdc).  $f_{xx}(\omega)$  and  $f_{yy}(\omega)$  are the univariate spectrum estimates for the two series as obtained using nag\_tsa\_spectrum\_univar\_cov (g13cac) or nag\_tsa\_spectrum\_univar (g13cbc). The same type and amount of smoothing should be used for these estimates, and this is specified by the degrees of freedom and bandwidth values which are passed from the calls of nag\_tsa\_spectrum\_univar\_cov (g13cac) or nag\_tsa\_spectrum\_univar (g13cbc).

Upper and lower 95% confidence limits for the cross amplitude are given approximately by

$$A(\omega) \left[ 1 \pm \left( 1.96/\sqrt{d} \right) \sqrt{W(\omega)^{-1} + 1} \right],$$

except that a negative lower limit is reset to 0.0, in which case the approximation is rather poor. You are therefore particularly recommended to compare the coherency estimate  $W(\omega)$  with the critical value  $T$  derived from the upper 5% point of the  $F$ -distribution on  $(2, d-2)$  degrees of freedom:

$$T = \frac{2F}{d-2+2F}$$

where  $d$  is the degrees of freedom associated with the univariate spectrum estimates. The value of  $T$  is returned by the function.

The hypothesis that the series are unrelated at frequency  $\omega$ , i.e., that both the true cross amplitude and coherency are zero, may be rejected at the 5% level if  $W(\omega) > T$ . Tests at two frequencies separated by more than the bandwidth may be taken to be independent.

The confidence limits on  $A(\omega)$  are strictly appropriate only at frequencies for which the coherency is significant. The same applies to the confidence limits on  $W(\omega)$  which are however calculated at all frequencies using the approximation that  $\text{arctanh}(\sqrt{W(l)})$  is Normal with variance  $1/d$ .

## 4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

## 5 Arguments

- 1: **xg[ng]** – const double *Input*  
*On entry:* the **ng** univariate spectral estimates,  $f_{xx}(\omega)$ , for the  $x$  series.
- 2: **yg[ng]** – const double *Input*  
*On entry:* the **ng** univariate spectral estimates,  $f_{yy}(\omega)$ , for the  $y$  series.
- 3: **xyg[ng]** – const Complex *Input*  
*On entry:*  $f_{xy}(\omega)$ , the **ng** bivariate spectral estimates for the  $x$  and  $y$  series. The  $x$  series leads the  $y$  series.  
**Note:** the two univariate and the bivariate spectra must each have been calculated using the same amount of smoothing. The frequency width and the shape of the window and the frequency division of the spectral estimates must be the same. The spectral estimates and statistics must also be unlogged.
- 4: **ng** – Integer *Input*  
*On entry:* the number of spectral estimates in each of the arrays **xg**, **yg** and **xyg**. It is also the number of cross amplitude spectral and squared coherency estimates.  
*Constraint:* **ng**  $\geq 1$ .
- 5: **stats[4]** – const double *Input*  
*On entry:* the 4 associated statistics for the univariate spectral estimates for the  $x$  and  $y$  series. **stats**[0] contains the degrees of freedom, **stats**[1] and **stats**[2] contain the lower and upper bound multiplying factors respectively and **stats**[3] contains the bandwidth.  
*Constraints:*

$$\begin{aligned} \mathbf{stats}[0] &\geq 3.0; \\ 0.0 < \mathbf{stats}[1] &\leq 1.0; \\ \mathbf{stats}[2] &\geq 1.0. \end{aligned}$$
- 6: **ca[ng]** – double *Output*  
*On exit:* the **ng** cross amplitude spectral estimates  $\hat{A}(\omega)$  at each frequency of  $\omega$ .
- 7: **calw[ng]** – double *Output*  
*On exit:* the **ng** lower bounds for the **ng** cross amplitude spectral estimates.
- 8: **caup[ng]** – double *Output*  
*On exit:* the **ng** upper bounds for the **ng** cross amplitude spectral estimates.
- 9: **t** – double \* *Output*  
*On exit:* the critical value for the significance of the squared coherency,  $T$ .

- 10: **sc[ng]** – double *Output*  
*On exit:* the **ng** squared coherency estimates,  $\hat{W}(\omega)$  at each frequency  $\omega$ .
- 11: **sclw[ng]** – double *Output*  
*On exit:* the **ng** lower bounds for the **ng** squared coherency estimates.
- 12: **scup[ng]** – double *Output*  
*On exit:* the **ng** upper bounds for the **ng** squared coherency estimates.
- 13: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BIVAR\_SPECTRAL\_ESTIM\_ZERO

A bivariate spectral estimate is zero.

For this frequency the cross amplitude spectrum is set to zero, and the contributions to the impulse response function and its standard error are set to zero.

### NE\_INT\_ARG\_LT

On entry, **ng** =  $\langle value \rangle$ .

Constraint: **ng**  $\geq 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

### NE\_REAL\_ARG\_GT

On entry, **stats**[1] must not be greater than 1.0: **stats**[1] =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_LE

On entry, **stats**[1] must not be less than or equal to 0.0: **stats**[1] =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_LT

On entry, **stats**[0] must not be less than 3.0: **stats**[0] =  $\langle value \rangle$ .

On entry, **stats**[2] must not be less than 1.0: **stats**[2] =  $\langle value \rangle$ .

### NE\_SQUARED\_FREQ\_GT\_ONE

A calculated value of the squared coherency exceeds one.

For this frequency the squared coherency is reset to one with the result that the cross amplitude spectrum is zero and the contribution to the impulse response function at this frequency is zero.

### NE\_UNIVAR\_SPECTRAL\_ESTIM\_NEG

A bivariate spectral estimate is negative.

For this frequency the cross amplitude spectrum is set to zero, and the contributions to the impulse response function and its standard error are set to zero.

## NE\_UNIVAR\_SPECTRAL\_ESTIM\_ZERO

A bivariate spectral estimate is zero.

For this frequency the cross amplitude spectrum is set to zero, and the contributions to the impulse response function and its standard error are set to zero.

## 7 Accuracy

All computations are very stable and yield good accuracy.

## 8 Parallelism and Performance

nag\_tsa\_cross\_spectrum\_bivar (g13cec) is not threaded in any implementation.

## 9 Further Comments

The time taken by nag\_tsa\_cross\_spectrum\_bivar (g13cec) is approximately proportional to **ng**.

## 10 Example

The example program reads the set of univariate spectrum statistics, the 2 univariate spectra and the cross spectrum at a frequency division of  $\frac{2\pi}{20}$  for a pair of time series. It calls nag\_tsa\_cross\_spectrum\_bivar (g13cec) to calculate the cross amplitude spectrum and squared coherency and their bounds and prints the results.

### 10.1 Program Text

```
/* nag_tsa_cross_spectrum_bivar (g13cec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagg13.h>

#define L      80
#define KC     8*L
#define NGMAX  KC
#define NXYMAX 300

int main(void)
{
    Complex *xyg = 0;
    Integer exit_status = 0, i, is, j, kc = KC, l = L, mw, ng, nxy;
    NagError fail;
    double *ca = 0, *calw = 0, *caup = 0, pw, pxy, *sc = 0, *sclw = 0;
    double *scup = 0, stats[4], t, *x = 0, *xg = 0, *y = 0, *yg = 0;
    INIT_FAIL(fail);

    printf("nag_tsa_cross_spectrum_bivar (g13cec) Example Program Results\n");
```

```

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &nxy);
#else
    scanf("%" NAG_IFMT " ", &nxy);
#endif

if (nxy > 0 && nxy <= NXYMAX) {
    if (!(x = NAG_ALLOC(KC, double)) ||
        !(y = NAG_ALLOC(KC, double)) ||
        !(ca = NAG_ALLOC(NGMAX, double)) ||
        !(calw = NAG_ALLOC(NGMAX, double)) ||
        !(caup = NAG_ALLOC(NGMAX, double)) ||
        !(sc = NAG_ALLOC(NGMAX, double)) ||
        !(sclw = NAG_ALLOC(NGMAX, double)) ||
        !(scup = NAG_ALLOC(NGMAX, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    for (i = 1; i <= nxy; ++i)
#ifdef _WIN32
        scanf_s("%lf ", &x[i - 1]);
#else
        scanf("%lf ", &x[i - 1]);
#endif
    for (i = 1; i <= nxy; ++i)
#ifdef _WIN32
        scanf_s("%lf ", &y[i - 1]);
#else
        scanf("%lf ", &y[i - 1]);
#endif

    /* Set parameters for call to nag_tsa_spectrum_univar (g13cbc) and g13cdc
     * with mean correction and 10 percent taper
     */
    pxy = 0.1;
    /* Window shape parameter and zero covariance at lag 16 */
    pw = 0.5;
    mw = 16;
    /* Alignment shift of 3 */
    is = 3;

    /* Obtain univariate spectrum for the x and the y series */
    /* nag_tsa_spectrum_univar (g13cbc).
     * Univariate time series, smoothed sample spectrum using
     * spectral smoothing by the trapezium frequency (Daniell)
     * window
     */
    nag_tsa_spectrum_univar(nxy, Nag_Mean, pxy, mw, pw, 1, kc, Nag_Unlogged,
                           x, &xg, &ng, stats, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_spectrum_univar (g13cbc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
    /* nag_tsa_spectrum_univar (g13cbc), see above. */
    nag_tsa_spectrum_univar(nxy, Nag_Mean, pxy, mw, pw, 1, kc, Nag_Unlogged,
                           y, &yg, &ng, stats, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_spectrum_univar (g13cbc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
}

```

```

    }

    /* Obtain cross spectrum of the bivariate series */
    /* nag_tsa_spectrum_bivar (g13cdc).
    * Multivariate time series, smoothed sample cross spectrum
    * using spectral smoothing by the trapezium frequency
    * (Daniell) window
    */
    nag_tsa_spectrum_bivar(nxy, Nag_Mean, pxy, mw, is, pw, l, kc, x, y, &xyg,
                          &ng, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_spectrum_bivar (g13cdc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }

    /* nag_tsa_cross_spectrum_bivar (g13cec).
    * Multivariate time series, cross amplitude spectrum,
    * squared coherency, bounds, univariate and bivariate
    * (cross) spectra
    */
    nag_tsa_cross_spectrum_bivar(xg, yg, xyg, ng, stats, ca, calw, caup, &t,
                                sc, sclw, scup, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_cross_spectrum_bivar (g13cec).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }

    printf("\n");
    printf("      Cross amplitude spectrum\n\n");
    printf("      Lower      Upper\n");
    printf("      Value      bound      bound\n\n");
    for (j = 1; j <= ng; ++j)
        printf(" %5" NAG_IFMT "%10.4f%10.4f%10.4f\n",
              j - 1, ca[j - 1], calw[j - 1], caup[j - 1]);
    printf("\n");
    printf(" Squared coherency test statistic =%12.4f\n\n", t);
    printf(" Squared coherency\n\n");
    printf("      Lower      Upper\n");
    printf("      Value      bound      bound\n\n");
    for (j = 1; j <= ng; ++j)
        printf(" %5" NAG_IFMT "%10.4f%10.4f%10.4f\n",
              j - 1, sc[j - 1], sclw[j - 1], scup[j - 1]);
    }
    NAG_FREE(xg);
    NAG_FREE(yg);
    NAG_FREE(xyg);
END:
    NAG_FREE(x);
    NAG_FREE(y);
    NAG_FREE(ca);
    NAG_FREE(calw);
    NAG_FREE(caup);
    NAG_FREE(sc);
    NAG_FREE(sclw);
    NAG_FREE(scup);
    return exit_status;
}

```

## 10.2 Program Data

nag\_tsa\_cross\_spectrum\_bivar (g13cec) Example Program Data

```

296
-0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
 0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
 0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767

```

```

1.608 1.265 0.790 0.360 0.115 0.088 0.331 0.645
0.960 1.409 2.670 2.834 2.812 2.483 1.929 1.485
1.214 1.239 1.608 1.905 2.023 1.815 0.535 0.122
0.009 0.164 0.671 1.019 1.146 1.155 1.112 1.121
1.223 1.257 1.157 0.913 0.620 0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524 0.040 0.788 0.943 0.930 1.006 1.137 1.198
1.054 0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007 0.254 0.330 0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271 0.000 0.403
0.841 1.285 1.607 1.746 1.683 1.485 0.993 0.648
0.577 0.577 0.632 0.747 0.999 0.993 0.968 0.790
0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049 0.060
0.161 0.301 0.517 0.566 0.560 0.573 0.592 0.671
0.933 1.337 1.460 1.353 0.772 0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676 0.033 0.556 0.643 0.484
0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
0.063 0.084 0.000 0.001 0.209 0.556 0.782 0.858
0.918 0.862 0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395 0.185 0.662 0.709 0.605 0.501 0.603
0.943 1.223 1.249 0.824 0.102 0.025 0.382 0.922
1.032 0.866 0.527 0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033 0.102 0.251
0.280 0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
0.034 0.204 0.253 0.195 0.131 0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

### 10.3 Program Results

nag\_tsa\_cross\_spectrum\_bivar (g13cec) Example Program Results

Cross amplitude spectrum

	Value	Lower bound	Upper bound
0	6.1563	3.6901	10.2708
1	5.9415	3.5602	9.9155
2	4.3034	2.5797	7.1791
3	2.6086	1.5642	4.3504
4	1.8249	1.0918	3.0502
5	1.1865	0.7060	1.9941
6	0.9616	0.5694	1.6239
7	0.6893	0.4080	1.1644

8	0.4629	0.2728	0.7855
9	0.2398	0.1407	0.4087
10	0.1199	0.0698	0.2060
11	0.0769	0.0441	0.1342
12	0.0463	0.0262	0.0819
13	0.0301	0.0166	0.0545
14	0.0158	0.0083	0.0301
15	0.0089	0.0046	0.0175
16	0.0050	0.0024	0.0103
17	0.0016	0.0007	0.0040
18	0.0004	0.0001	0.0024
19	0.0004	0.0001	0.0018
20	0.0003	0.0001	0.0016
21	0.0003	0.0000	0.0015
22	0.0004	0.0001	0.0018
23	0.0003	0.0000	0.0026
24	0.0002	0.0000	0.0028
25	0.0004	0.0001	0.0015
26	0.0003	0.0001	0.0010
27	0.0001	0.0000	0.0018
28	0.0002	0.0000	0.0013
29	0.0003	0.0001	0.0010
30	0.0004	0.0001	0.0010
31	0.0002	0.0001	0.0009
32	0.0001	0.0000	0.0022
33	0.0000	0.0000	4.4716
34	0.0001	0.0000	0.0007
35	0.0002	0.0000	0.0006
36	0.0002	0.0000	0.0006
37	0.0001	0.0000	0.0006
38	0.0001	0.0000	0.0007
39	0.0001	0.0000	0.0012
40	0.0001	0.0000	0.0012

Squared coherency test statistic = 0.1926

Squared coherency

	Value	Lower bound	Upper bound
0	0.9562	0.9124	0.9783
1	0.9539	0.9079	0.9772
2	0.9567	0.9134	0.9786
3	0.9591	0.9181	0.9798
4	0.9428	0.8864	0.9716
5	0.9048	0.8148	0.9523
6	0.8738	0.7586	0.9362
7	0.8715	0.7545	0.9350
8	0.8453	0.7087	0.9212
9	0.8198	0.6654	0.9075
10	0.7770	0.5956	0.8841
11	0.7033	0.4838	0.8422
12	0.6541	0.4152	0.8131
13	0.5702	0.3090	0.7609
14	0.4474	0.1786	0.6776
15	0.3945	0.1318	0.6385
16	0.3260	0.0802	0.5844
17	0.1887	0.0115	0.4580
18	0.0390	0.0000	0.2566
19	0.0618	0.0000	0.2975
20	0.0524	0.0000	0.2815
21	0.0428	0.0000	0.2639
22	0.0578	0.0000	0.2907
23	0.0282	0.0000	0.2337
24	0.0220	0.0000	0.2188
25	0.0807	0.0000	0.3268
26	0.0754	0.0000	0.3188
27	0.0167	0.0000	0.2047
28	0.0331	0.0000	0.2445
29	0.0777	0.0000	0.3223



30	0.1414	0.0014	0.4059
31	0.0739	0.0000	0.3166
32	0.0107	0.0000	0.1861
33	0.0008	0.0000	0.1358
34	0.0488	0.0000	0.2750
35	0.0760	0.0000	0.3198
36	0.0738	0.0000	0.3165
37	0.0719	0.0000	0.3135
38	0.0438	0.0000	0.2657
39	0.0168	0.0000	0.2049
40	0.0183	0.0000	0.2091

---