

# NAG Library Function Document

## nag\_tsa\_spectrum\_bivar (g13cdc)

### 1 Purpose

nag\_tsa\_spectrum\_bivar (g13cdc) calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

### 2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_spectrum_bivar (Integer nxy, NagMeanOrTrend mt_correction,
    double pxy, Integer mw, Integer is, double pw, Integer l, Integer kc,
    const double x[], const double y[], Complex **g, Integer *ng,
    NagError *fail)
```

### 3 Description

The supplied time series may be mean and trend corrected and tapered as in the description of nag\_tsa\_spectrum\_univar (g13cbc) before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^*(\omega) = \frac{1}{2\pi n} \left\{ \sum_{t=1}^n y_t \exp(i\omega t) \right\} \times \left\{ \sum_{t=1}^n x_t \exp(-i\omega t) \right\}$$

for frequency values  $\omega_j = \frac{2\pi j}{K}$ ,  $0 \leq \omega_j \leq \pi$ .

A correction is made for bias due to any tapering.

As in the description of nag\_tsa\_spectrum\_univar (g13cbc) for univariate frequency window smoothing, the smoothed spectrum is returned at a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}, l = 0, 1, \dots, [L/2]$$

where  $[ ]$  denotes the integer part.

Its real part or co-spectrum  $cf(\nu_l)$ , and imaginary part or quadrature spectrum  $qf(\nu_l)$  are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \pi/M} \tilde{w}_k f_{xy}^*(\nu_l + \omega_k)$$

where the weights  $\tilde{w}_k$  are similar to the weights  $w_k$  defined for nag\_tsa\_spectrum\_univar (g13cbc), but allow for an implicit alignment shift  $S$  between the series:

$$\tilde{w}_k = w_k \exp(-2\pi i S k / L).$$

It is recommended that  $S$  is chosen as the lag  $k$  at which the cross-covariances  $c_{xy}(k)$  peak, so as to minimize bias.

If no smoothing is required, the integer  $M$  which determines the frequency window width  $\frac{2\pi}{M}$ , should be set to  $n$ .

The bandwidth of the estimates will normally have been calculated in a previous call of nag\_tsa\_spectrum\_univar (g13cbc) for estimating the univariate spectra of  $y_t$  and  $x_t$ .

## 4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

## 5 Arguments

- 1:    **nxy** – Integer *Input*  
*On entry:* the length of the time series  $x$  and  $y$ ,  $n$ .  
*Constraint:* **nxy**  $\geq 1$ .
  
- 2:    **mt\_correction** – NagMeanOrTrend *Input*  
*On entry:* whether the data are to be initially mean or trend corrected.  
**mt\_correction** = Nag\_NoCorrection for no correction, **mt\_correction** = Nag\_Mean for mean correction, **mt\_correction** = Nag\_Trend for trend correction.  
*Constraint:* **mt\_correction** = Nag\_NoCorrection, Nag\_Mean or Nag\_Trend.
  
- 3:    **pxy** – double *Input*  
*On entry:* the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.  
A value of 0.0 implies no tapering.  
*Constraint:*  $0.0 \leq \mathbf{pxy} \leq 1.0$ .
  
- 4:    **mw** – Integer *Input*  
*On entry:* the frequency width,  $M$ , of the smoothing window as  $2\pi/M$ .  
A value of  $n$  implies that no smoothing is to be carried out.  
*Constraint:*  $1 \leq \mathbf{mw} \leq \mathbf{nxy}$ .
  
- 5:    **is** – Integer *Input*  
*On entry:* the alignment shift,  $S$ , between the  $x$  and  $y$  series. If  $x$  leads  $y$ , the shift is positive.  
*Constraint:*  $-1 < \mathbf{is} < 1$ .
  
- 6:    **pw** – double *Input*  
*On entry:* the shape argument,  $p$ , of the trapezium frequency window.  
A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.  
If **mw** = **nxy** (i.e., no smoothing is carried out) then **pw** is not used.  
*Constraint:*  $0.0 \leq \mathbf{pw} \leq 1.0$  if **mw**  $\neq$  **nxy**.
  
- 7:    **l** – Integer *Input*  
*On entry:* the frequency division,  $L$ , of smoothed cross spectral estimates as  $2\pi/L$ .  
*Constraint:* **l**  $\geq 1$ .  
**l** must be a factor of **kc** (see below).
  
- 8:    **kc** – Integer *Input*  
*On entry:* the order of the fast Fourier transform (FFT) used to calculate the spectral estimates.  
**kc** should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq 2n$ , provided  $m \leq 20$ .

*Constraints:*

**kc**  $\geq 2 \times \mathbf{nxy}$ ;

**kc** must be a multiple of **l**. The largest prime factor of **kc** must not exceed 19, and the total number of prime factors of **kc**, counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

- 9: **x[kc]** – const double *Input*  
*On entry:* the **nxy** data points of the *x* series.
- 10: **y[kc]** – const double *Input*  
*On entry:* the **nxy** data points of the *y* series.
- 11: **g** – Complex \*\* *Output*  
*On exit:* the complex vector which contains the **ng** cross spectral estimates in elements **g[0]** to **g[ng – 1]**. The *y* series leads the *x* series.  
 The memory for this vector is allocated internally. If no memory is allocated to **g** (e.g., when an input error is detected) then **g** will be **NULL** on return. If repeated calls to this function are required then **NAG\_FREE** should be used to free the memory in between calls.
- 12: **ng** – Integer \* *Output*  
*On exit:* the number of spectral estimates,  $[L/2] + 1$ , whose separate parts are held in **g**.
- 13: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_2\_INT\_ARG\_CONS

On entry, **kc** =  $\langle value \rangle$  while **l** =  $\langle value \rangle$ . These arguments must satisfy **kc%l**  $\neq 0$  when **l**  $> 0$ .

On entry, **kc** =  $\langle value \rangle$  while **nxy** =  $\langle value \rangle$ . These arguments must satisfy **kc**  $\geq 2 * \mathbf{nxy}$  when **nxy**  $> 0$ .

On entry, **l** =  $\langle value \rangle$  while **is** =  $\langle value \rangle$ . These arguments must satisfy  $\|\mathbf{is}\| < \mathbf{l}$  when **l**  $> 0$ .

### NE\_2\_INT\_ARG\_GT

On entry, **mw** =  $\langle value \rangle$  while **nxy** =  $\langle value \rangle$ . These arguments must satisfy **mw**  $\leq \mathbf{nxy}$ .

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument **mt\_correction** had an illegal value.

### NE\_FACTOR\_GT

At least one of the prime factors of **kc** is greater than 19.

### NE\_INT\_ARG\_LT

On entry, **l** =  $\langle value \rangle$ .

Constraint: **l**  $\geq 1$ .

On entry, **mw** =  $\langle value \rangle$ .

Constraint: **mw**  $\geq 1$ .

On entry, **nxy** =  $\langle value \rangle$ .

Constraint: **nxy**  $\geq 1$ .

## NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

## NE\_REAL\_ARG\_GT

On entry, **pw** must not be greater than 1.0: **pw** =  $\langle value \rangle$ .

On entry, **pxy** must not be greater than 1.0: **pxy** =  $\langle value \rangle$ .

## NE\_REAL\_ARG\_LT

On entry, **pw** must not be less than 0.0: **pw** =  $\langle value \rangle$ .

On entry, **pxy** must not be less than 0.0: **pxy** =  $\langle value \rangle$ .

## NE\_TOO\_MANY\_FACTORS

**kc** has more than 20 prime factors.

## 7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

## 8 Parallelism and Performance

nag\_tsa\_spectrum\_bivar (g13cdc) is not threaded in any implementation.

## 9 Further Comments

nag\_tsa\_spectrum\_bivar (g13cdc) carries out an FFT of length **kc** to calculate the sample cross spectrum. The time taken by the function for this is approximately proportional to  $\mathbf{kc} \times \log(\mathbf{kc})$  (but see function document nag\_sum\_fft\_realherm\_1d (c06pac) for further details).

## 10 Example

The example program reads 2 time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a  $2\pi/16$  frequency width of smoothing window, a window shape argument of 0.5 and an alignment shift of 3. It then calls nag\_tsa\_spectrum\_bivar (g13cdc) to calculate the smoothed sample cross spectrum and prints the results.

### 10.1 Program Text

```
/* nag_tsa_spectrum_bivar (g13cdc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 *
 */

#include <nag.h>
#include <stdio.h>
```

```

#include <nag_stdlib.h>
#include <naga02.h>
#include <nagg13.h>

#define L      80
#define KC     8*L
#define NXYMAX 300

int main(void)
{
    Complex *g;
    Integer exit_status = 0, i, is, j, kc = KC, l = L, mw, ng, nxy;
    NagError fail;
    double pw, pxy, *x = 0, *y = 0;

    INIT_FAIL(fail);

    printf("nag_tsa_spectrum_bivar (g13cdc) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &nxy);
#else
    scanf("%" NAG_IFMT " ", &nxy);
#endif
    if (nxy > 0 && nxy <= NXYMAX) {
        if (!(x = NAG_ALLOC(KC, double)) || !(y = NAG_ALLOC(KC, double)))
        {
            printf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
        for (i = 1; i <= nxy; ++i)
#ifdef _WIN32
            scanf_s("%lf ", &x[i - 1]);
#else
            scanf("%lf ", &x[i - 1]);
#endif
        for (i = 1; i <= nxy; ++i)
#ifdef _WIN32
            scanf_s("%lf ", &y[i - 1]);
#else
            scanf("%lf ", &y[i - 1]);
#endif
        /* Set parameters for call to nag_tsa_spectrum_bivar (g13cdc) */
        /* Mean correction and 10 percent taper */
        pxy = 0.1;
        /* Window shape parameter and zero covariance at lag 16 */
        pw = 0.5;
        mw = 16;
        /* Alignment shift of 3 */
        is = 3;

        /* nag_tsa_spectrum_bivar (g13cdc).
         * Multivariate time series, smoothed sample cross spectrum
         * using spectral smoothing by the trapezium frequency
         * (Daniell) window
         */
        nag_tsa_spectrum_bivar(nxy, Nag_Mean, pxy, mw, is, pw, l, kc, x, y, &g,
                               &ng, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_tsa_spectrum_bivar (g13cdc).\n%s\n",
                   fail.message);
        }
    }
}

```

```

        exit_status = 1;
        goto END;
    }

    printf("\n
    printf("\n      Real   Imaginary      Returned sample spectrum\n");
    printf("      Real   Imaginary\n");
    printf("      part   part           part   part   "
    printf("      part   part\n\n");
    for (j = 1; j <= ng; ++j)
        printf("%5" NAG_IFMT "%8.4f%9.4f%s",
            /* nag_complex_real (a02bbc).
            * Real part of a complex number
            */
            j, nag_complex_real(g[j - 1]), a02bcc(g[j - 1]),
            (j % 3 == 0 ? "\n" : ""));
    printf("\n");
    NAG_FREE(g);
}
END:
    NAG_FREE(x);
    NAG_FREE(y);
    return exit_status;
}

```

## 10.2 Program Data

nag\_tsa\_spectrum\_bivar (g13cdc) Example Program Data  
296

```

-0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
 0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
 0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
 1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
 0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
 1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
 0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
 1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
 1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
 0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
 0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
 0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
 0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
 0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
 0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
 0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
 0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
 0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
 1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
 0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
 0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5

```

```

50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

### 10.3 Program Results

nag\_tsa\_spectrum\_bivar (g13cdc) Example Program Results

Returned sample spectrum

	Real part	Imaginary part		Real part	Imaginary part		Real part	Imaginary part
1	-6.1563	0.0000	2	-5.5905	-2.0119	3	-3.2711	-2.7963
4	-1.1803	-2.3264	5	-0.2061	-1.8132	6	0.3434	-1.1357
7	0.6200	-0.7351	8	0.5967	-0.3449	9	0.4523	-0.0984
10	0.2391	0.0177	11	0.1129	0.0402	12	0.0564	0.0523
13	0.0134	0.0443	14	-0.0032	0.0299	15	-0.0057	0.0148
16	-0.0057	0.0069	17	-0.0033	0.0038	18	-0.0011	0.0012
19	-0.0004	0.0001	20	-0.0004	0.0002	21	-0.0003	0.0001
22	-0.0001	0.0002	23	-0.0002	0.0003	24	-0.0002	0.0002
25	-0.0002	0.0000	26	-0.0004	0.0000	27	-0.0002	-0.0002
28	-0.0001	-0.0000	29	-0.0001	0.0002	30	-0.0001	0.0002
31	-0.0002	0.0003	32	-0.0002	0.0001	33	-0.0001	0.0000
34	-0.0000	-0.0000	35	0.0000	-0.0001	36	0.0001	-0.0001
37	0.0001	-0.0001	38	0.0001	-0.0001	39	0.0000	-0.0001
40	0.0000	-0.0001	41	0.0001	0.0000			

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