

NAG Library Function Document

nag_tsa_multi_inp_model_estim (g13bec)

1 Purpose

nag_tsa_multi_inp_model_estim (g13bec) fits a time series model to one output series relating it to any input series with a choice of three different estimation criteria – nonlinear least squares, exact likelihood and marginal likelihood. When no input series are present, nag_tsa_multi_inp_model_estim (g13bec) fits a univariate ARIMA model.

2 Specification

```
#include <nag.h>
#include <naggl3.h>

void nag_tsa_multi_inp_model_estim (Nag_ArimaOrder *arimav, Integer nseries,
    Nag_TransfOrder *transfv, double para[], Integer npara, Integer nxy,
    const double xxy[], Integer tdxy, double sd[], double *rss,
    double *objf, double *df, Nag_G13_Opt *options, NagError *fail)
```

3 Description

3.1 The Multi-input Model

The output series y_t , for $t = 1, 2, \dots, n$, is assumed to be the sum of (unobserved) components $z_{i,t}$ which are due respectively to the inputs $x_{i,t}$, for $i = 1, 2, \dots, m$.

Thus $y_t = z_{1,t} + \dots + z_{m,t} + n_t$ where n_t is the error, or output noise component.

A typical component z_t may be either:

- (a) A simple regression component, $z_t = \omega x_t$ (here x_t is called a simple input) or
- (b) A transfer function model component which allows for the effect of lagged values of the variable, related to x_t by

$$z_t = \delta_1 z_{t-1} + \delta_2 z_{t-2} + \dots + \delta_p z_{t-p} + \omega_0 x_{t-b} - \omega_1 x_{t-b-1} - \dots - \omega_q x_{t-b-q}.$$

The noise n_t is assumed to follow a (possibly seasonal) ARIMA model, i.e., may be represented in terms of an uncorrelated series, a_t , by the hierarchy of equations:

$$\begin{array}{lcl} \nabla^d \nabla_s^D n_t & & c + w_t \\ w_t & \Phi_1 w_{t-s} + \Phi_2 w_{t-2s} + \dots + \Phi_P w_{t-Ps} + e_t - \Theta_1 e_{t-s} - \Theta_2 e_{t-2s} - \dots - \Theta_Q e_{t-Qs} & \\ e_t & \phi_1 e_{t-1} + \phi_2 e_{t-2} + \dots + \phi_p e_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} & \end{array}$$

Note: the orders p, q appearing in each of the transfer function models and the ARIMA model are not necessarily the same; $\nabla^d \nabla_s^D n_t$ is the result of applying non-seasonal differencing of order d and seasonal differencing of seasonality s and order D to the series n_t , the differenced series is then of length $N = n - d - s \times D$; the constant term argument c may optionally be held fixed at its initial value (usually, but not necessarily zero) rather than being estimated.

For the purpose of defining an estimation criterion it is assumed that the series a_t is a sequence of independent Normal variates having mean 0 and variance σ_a^2 . An allowance has to be made for the effects of unobserved data prior to the observation period. For the noise component an allowance is always made using a form of backforecasting.

For each transfer function input, you have to decide what values are to be assumed for the pre-period terms $z_0, z_{-1}, \dots, z_{1-p}$ and $x_0, x_{-1}, \dots, x_{1-b-q}$ which are in theory necessary to re-create the component series z_1, z_2, \dots, z_n , during the estimation procedure.

The first choice is to assume that all these values are zero. In this case in order to avoid undesirable transient distortion of the early values z_1, z_2, \dots , you are advised first to correct the input series x_t by subtracting from all the terms a suitable constant to make the early values x_1, x_2, \dots , close to zero. The series mean \bar{x} is one possibility, but for a series with strong trend, the constant might be simply x_1 .

The second choice is to treat the unknown pre-period terms as nuisance arguments and estimate them along with the other arguments. This choice should be used with caution. For example, if $p = 1$ and $b = q = 0$, it is equivalent to fitting to the data a decaying geometric curve of the form $A\delta^t$, for $t = 1, 2, \dots$, along with the other inputs, this being the form of the transient. If the output y_t contains a strong trend of this form, which is not otherwise represented in the model, it will have a tendency to influence the estimate of δ away from the value appropriate to the transfer function model.

In most applications the first choice should be adequate, with the option possibly being used as a refinement at the end of the modelling process. The number of nuisance arguments is then $\max(p, b + q)$, with a corresponding loss of degrees of freedom in the residuals. If you align the input x_t with the output by using in its place the shifted series x_{t-b} , then setting $b = 0$ in the transfer function model, there is some improvement in efficiency. On some occasions when the model contains two or more inputs, each with estimation of pre-period nuisance arguments, these arguments may be co-linear and lead to failure of the function. The option must then be ‘switched off’ for one or more inputs.

3.2 The Estimation Criterion

This is a measure of how well a proposed set of arguments in the transfer function and noise ARIMA models, matches the data. The estimation function searches for argument values which minimize this criterion. For a proposed set of argument values it is derived by calculating:

- (i) the components $z_{1,t}, z_{2,t}, \dots, z_{m,t}$ as the responses to the input series $x_{1,t}, x_{2,t}, \dots, x_{m,t}$ using the equations (a) or (b) above,
- (ii) the discrepancy between the output and the sum of these components, as the noise

$$n_t = y_t - (z_{1,t} + z_{2,t} + \dots + z_{m,t}),$$

- (iii) the residual series a_t from n_t by reversing the recursive equations (c), (d) and (e) above.

This last step again requires treatment of the effect of unknown pre-period values of n_t and other terms in the equations regenerating a_t . One approach is to use a sum of squares function as the estimation criteria, which is equivalent to taking the infinite set of past values $n_0, n_{-1}, n_{-2}, \dots$, as (linear) nuisance arguments. There is no loss of degrees of freedom however, because the sum of squares function S may be expressed as including the corresponding set of past residuals – see Box and Jenkins (1976) page 273, who prove that

$$S = \sum_{-\infty}^n a_t^2.$$

The function $D = S$ is the first of the three possible criteria, and is quite adequate for moderate to long series with no seasonal arguments. The second is the exact likelihood criterion which considers the past set $n_0, n_{-1}, n_{-2}, \dots$, not as simple nuisance arguments, but as unobserved random variables with known distribution. Calculation of the likelihood of the observed set n_1, n_2, \dots, n_n requires theoretical integration over the range of the past set. Fortunately this yields a criterion of the form $D = M \times S$ (whose minimization is equivalent to maximizing the exact likelihood of the data), where S is exactly as before, and the multiplier M is a function calculated from the ARIMA model arguments. The value of M is always ≥ 1 , and M tends to 1 for any fixed argument set as the sample size n tends to ∞ . There is a moderate computational overhead in using this option, but its use avoids appreciable bias in the ARIMA model arguments and yields a better conditioned estimation problem.

The third criterion of marginal likelihood treats the coefficients of the simple inputs in a manner analogous to that given to the past set $n_0, n_{-1}, n_{-2}, \dots$. These coefficients, together with the constant term c used to represent the mean of w_t , are in effect treated as random variables with highly dispersed distributions. This leads to the criterion $D = M \times S$ again, but with a different value of M which now depends on the simple input series values x_t . In the presence of a moderate to large number of simple inputs, the marginal likelihood criterion can counteract bias in the ARIMA model arguments which is caused by estimation of the simple inputs. This is particularly important in relatively short series.

`nag_tsa_multi_inp_model_estim` (g13bec) can be used with no input series present, to estimate a univariate ARIMA model for the output alone. The marginal likelihood criterion is then distinct from exact likelihood only if a constant term is being estimated in the model, because this is treated as an implicit simple input.

3.3 The Estimation Procedure

This is the minimization of the estimation criterion or objective function D (for deviance). The function uses an extension of the algorithm of Marquardt (1963). The step size in the minimization is inversely related to an argument α , which is increased or decreased by a factor β at successive iterations, depending on the progress of the minimization. Convergence is deemed to have occurred if the fractional reduction of D in successive iterations is less than a value γ , while $\alpha < 1$.

Certain model arguments (in fact all excluding the ω 's) are subject to stability constraints which are checked throughout to within a specified tolerance multiple δ of machine accuracy. Using the least squares criterion, the minimization may halt prematurely when some arguments ‘stick’ at a constraint boundary. This can happen particularly with short seasonal series (with a small number of whole seasons). It will not happen using the exact likelihood criterion, although convergence to a point on the boundary may sometimes be rather slow, because the criterion function may be very flat in such a region. There is also a smaller risk of a premature halt at a constraint boundary when marginal likelihood is used.

A positive, or zero number of iterations can be specified. In either case, the value D of the objective function at iteration zero is computed at the initial argument values, except for the estimation of any pre-period terms for the input series, backforecasts for the noise series, and the coefficients of any simple inputs, and the constant term (unless this is held fixed).

At any later iteration, the value of D is computed after re-estimation of the backforecasts to their optimal values, corresponding to the model arguments presented at that iteration. This is not true for any pre-period terms for the input series which, although they are updated from the previous iteration, may not be precisely optimal for the argument values presented, unless convergence of those arguments has occurred. However, in the case of marginal likelihood being specified, the coefficients of the simple inputs and the constant term are also re-estimated together with the backforecasts at each iteration, to values which are optimal for the other argument values presented.

3.4 Further Results

The residual variance is taken as $erv = \frac{S}{df}$ where $df = N -$ (total number of arguments estimated), is the residual degrees of freedom (for definition of S see Section 3.2 and for definition of N see Section 3.1). The pre-period nuisance arguments for the input series are included in the reduction of df , as is the constant if it is estimated.

The covariance matrix of the vector of model parameter estimates is given by

$$erv \times H^{-1}$$

where H is the linearised least squares matrix taken from the final iteration of the algorithm of Marquardt. From this expression are derived the vector of standard deviations, and the correlation matrix of parameter estimates. These are approximations which are only valid asymptotically, and must be treated with great caution when the parameter estimates are close to their constraint boundaries.

The residual series a_t is available upon completion of the iterations over the range $t = 1 + d + s \times D, \dots, n$ corresponding to the differenced noise series w_t .

Because of the algorithm used for backforecasting, these are only true residuals for $t \geq 1 + q + s \times Q - p - s \times P - d - s \times D$, provided this is positive. Estimation of pre-period terms for the inputs will also tend to reduce the magnitude of the early residuals, sometimes severely.

The model component series $z_{1,t}, \dots, z_{m,t}$ and n_t may optionally be returned in order to assess the effects of the various inputs on the output.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

Byng M Fitting a seasonal ARIMA model using the NAG C Library *NAG Technical Report TR3/09*
<http://www.nag.co.uk/technical-report-repository#tr0309>

Marquardt D W (1963) An algorithm for least squares estimation of nonlinear parameters *J. Soc. Indust. Appl. Math.* **11** 431

5 Arguments

1: **arimav** – Nag_ArimaOrder *

Pointer to structure of type Nag_ArimaOrder with the following members:

| | |
|-----------------------|--------------|
| p – Integer | |
| d – Integer | <i>Input</i> |
| q – Integer | <i>Input</i> |
| bigp – Integer | <i>Input</i> |
| bigd – Integer | <i>Input</i> |
| bigq – Integer | <i>Input</i> |
| s – Integer | <i>Input</i> |

On entry: these seven members of **arimav** must specify the orders vector (p, d, q, P, D, Q, s) , respectively, of the ARIMA model for the output noise component.

p, q, P and Q refer, respectively, to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) arguments.

d, D and s refer, respectively, to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

Constraints:

$$\begin{aligned}
 &p, d, q, P, D, Q, s \geq 0; \\
 &p + q + P + Q > 0; \\
 &s \neq 1; \\
 &\text{if } s = 0, P + D + Q = 0; \\
 &\text{if } s > 1, P + D + Q > 0; \\
 &d + s \times (P + D) \leq n; \\
 &p + d - q + s \times (P + D - Q) \leq n.
 \end{aligned}$$

2: **nseries** – Integer *Input*

On entry: the total number of input and output series. There may be any number of input series (including none), but always one output series.

Constraint: **nseries** > 1 if there are no arguments in the model (that is $p = q = P = Q = 0$ and **options.cfired** = Nag.TRUE), **nseries** ≥ 1 otherwise.

3: **transfv** – Nag_TransfOrder *

Pointer to structure of type Nag_TransfOrder with the following members:

| | |
|----------------------|---------------------|
| b – Integer * | <i>Input/Output</i> |
| q – Integer * | <i>Input/Output</i> |
| p – Integer * | |
| r – Integer * | <i>Input/Output</i> |

On entry/exit: before use these member pointers **must** be allocated memory by calling `nag_tsa_transf_orders` (g13byc) which allocates **nseries** – 1 elements to each pointer. The memory allocated to these pointers must be given the transfer function model orders b, q

and p of each of the input series. The order arguments for input series i are held in the i th element of the allocated memory for each pointer. **transfv**→**b**[$i - 1$] holds the value b_i , **transfv**→**q**[$i - 1$] holds the value q_i and **transfv**→**p**[$i - 1$] holds the value p_i . For a simple input, $b_i = q_i = p_i = 0$. **transfv**→**r**[$i - 1$] holds the value r_i , where $r_i = 1$ for a simple input, $r_i = 2$ for a transfer function input for which no allowance is to be made for pre-observation period effects, and $r_i = 3$ for a transfer function input for which pre-observation period effects will be treated by estimation of appropriate nuisance arguments. When $r_i = 1$, any nonzero contents of the i th element of **transfv**→**b**, **transfv**→**q** and **transfv**→**p** are ignored.

Constraint: **transfv**→**r**[$i - 1$] = 1, 2 or 3, for $i = 1, 2, \dots, \text{nseries} - 1$

The memory allocated to the members of **transfv** must be freed by a call to **nag_tsa_trans_free** (g13bzc)

- 4: **para**[**npara**] – double *Input/Output*
On entry: initial values of the multi-input model arguments. These are in order, firstly the ARIMA model arguments: p values of ϕ arguments, q values of θ arguments, P values of Φ arguments and Q values of Θ arguments. These are followed by initial values of the transfer function model arguments $\omega_0, \omega_1, \dots, \omega_{q_1}, \delta_1, \delta_2, \dots, \delta_{p_1}$ for the first of any input series and similarly for each subsequent input series. The final component of **para** is the initial value of the constant c , whether it is fixed or is to be estimated.
On exit: the latest values of the estimates of these arguments.
- 5: **npara** – Integer *Input*
On entry: the exact number of $\phi, \theta, \Phi, \Theta, \omega, \delta$ and c arguments.
Constraint: **npara** = $p + q + P + Q + \text{nseries} + \sum(p_i + q_i)$, the summation being over all the input series. (c must be included, whether fixed or estimated.)
- 6: **nxxxy** – Integer *Input*
On entry: the (common) length of the original, undifferenced input and output time series.
- 7: **xxxy**[**nxxxy** × **tdxxxy**] – const double *Input*
Note: the (i, j) th element of the matrix is stored in **xxxy**[($i - 1$) × **tdxxxy** + $j - 1$].
On entry: the columns of **xxxy** must contain the **nxxxy** original, undifferenced values of each of the input series, x_t , and the output series, y_t , in that order.
- 8: **tdxxxy** – Integer *Input*
On entry: the stride separating matrix column elements in the array **xxxy**.
Constraint: **tdxxxy** ≥ **nseries**.
- 9: **sd**[**npara**] – double *Output*
On exit: the **npara** values of the standard deviations corresponding to each of the arguments in **para**. When the constant is fixed its standard deviation is returned as zero. When the values of **para** are valid, the values of **sd** are usually also valid unless the function fails to invert the second derivative matrix in which case **fail** will have an exit value of NE_MAT_NOT_POS_DEF.
- 10: **rss** – double * *Output*
On exit: the residual sum of squares, S , at the latest set of valid parameter estimates.
- 11: **objf** – double * *Output*
On exit: the objective function, D , at the latest set of valid parameter estimates.

- 12: **df** – double * *Output*
On exit: the degrees of freedom associated with S .
- 13: **options** – Nag_G13_Opt * *Input/Output*
On entry/exit: a pointer to a structure of type Nag_G13_Opt whose members are optional parameters for nag_tsa_multi_inp_model_estim (g13bec). If the optional parameters are not required, then the null pointer, G13_DEFAULT, can be used in the function call to nag_tsa_multi_inp_model_estim (g13bec). Details of the optional parameters and their types are given below in Section 11.2.
- 14: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

If the intermediate results of optimization are written to a file using the optional parameter **options.outfile**, then NE_NOT_APPEND_FILE, NE_WRITE_ERROR and NE_NOT_CLOSE_FILE could also occur.

NE_2_INT_ARG_LT

On entry, **tdxxy** = $\langle value \rangle$ while **nseries** = $\langle value \rangle$. These arguments must satisfy **tdxxy** \geq **nseries**.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_ARIMA_TEST_FAILED

On entry, or during execution, one or more sets of the ARIMA (ϕ , θ , Φ or Θ) arguments do not satisfy the stationarity or invertibility test conditions.

NE_BAD_PARAM

On entry, argument **options.cfixed** had an illegal value.

On entry, argument **options.criteria** had an illegal value.

On entry, argument **options.print_level** had an illegal value.

NE_CONSTRAINT

General constraint: $\langle value \rangle$.

NE_DELTA_TEST_FAILED

On entry, or during execution, one or more sets of δ arguments do not satisfy the stationarity or invertibility test conditions.

NE_DIFORDER_LEN_INCONSIST

The orders of differencing specified in the structure arimav must satisfy **nxy** $>$ **arimav**→**d** + (**arimav**→**s** * **arimav**→**bigd**), **nxy** = $\langle value \rangle$, **arimav**→**d** = $\langle value \rangle$, **arimav**→**s** = $\langle value \rangle$, **arimav**→**bigd** = $\langle value \rangle$.

NE_G13_OPTIONS_NOT_INIT

On entry, the option structure, **options**, has not been initialized using nag_tsa_options_init (g13bxc).

NE_G13_ORDERS_NOT_INIT

On entry, the orders array structure, **transfv**, has not been successfully initialized using function `nag_tsa_transf_orders` (g13byc).

NE_INT_ARG_LT

On entry, **nseries** = $\langle value \rangle$.
Constraint: **nseries** ≥ 1 .

On entry, **options.gamma** = $\langle value \rangle$.
Constraint: **options.gamma** ≥ 0.0 .

On entry, **options.max_iter** = $\langle value \rangle$.
Constraint: **options.max_iter** ≥ 0 .

NE_INT_ARRAY_2

Value $\langle value \rangle$ given to **transfv.transfv**→**r**[$\langle value \rangle$] not valid. Correct range for elements of **transfv.transfv**→**r** is $1 \leq \text{transfv} \rightarrow \mathbf{r}[i] \leq 3$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_INVALID_NSER

On entry, **nseries** = 1 and there are no arguments in the model, i.e., ($p = q = P = Q = 0$ and **options.cfixed** = Nag.TRUE).

NE_ITER_FAIL_NIT

The function has failed to converge after **options.max_iter** iterations, where **options.max_iter** = $\langle value \rangle$. If steady decreases in the objective function, D , were monitored up to the point where this exit occurred, see the optional parameter **options.print_level**, then **options.max_iter** was probably set too small. If so the calculations should be restarted from the final point held in **para**.

NE_MAT_NOT_POS_DEF

Attempt to invert the second derivative matrix needed in the calculation of the covariance matrix of the parameter estimates has failed. The matrix is not positive definite, possibly due to rounding errors.

NE_NOT_APPEND_FILE

Cannot open file $\langle string \rangle$ for appending.

NE_NOT_CLOSE_FILE

Cannot close file $\langle string \rangle$.

NE_NPARA_MR_MT_INCONSIST

On entry, there is inconsistency between **npara** on the one hand and the elements in the orders structures, **arimav** and **transfv** on the other.

NE_NSER_INCONSIST

Value of **nseries** passed to `nag_tsa_transf_orders` (g13byc) was $\langle value \rangle$ which is not equal to the value $\langle value \rangle$ passed in this function.

NE_REAL_ARG_GE

On entry, **options.gamma** must not be greater than or equal to 1.0: **options.gamma** = $\langle value \rangle$.

NE_REAL_ARG_LE

On entry, **options.alpha** = $\langle value \rangle$.

Constraint: **options.alpha** > 0.0.

On entry, **options.beta** = $\langle value \rangle$.

Constraint: **options.beta** > 1.0.

NE_REAL_ARG_LT

On entry, **options.delta** = $\langle value \rangle$.

Constraint: **options.delta** \geq 1.0.

NE_SOLUTION_FAIL_CONV

Iterative refinement has failed to improve the solution of the equations giving the latest estimates of the arguments. This occurred because the matrix of the set of equations is too ill-conditioned.

NE_WRITE_ERROR

Error occurred when writing to file $\langle string \rangle$.

7 Accuracy

The computation used is believed to be stable.

8 Parallelism and Performance

nag_tsa_multi_inp_model_estim (g13bec) is not threaded in any implementation.

9 Further Comments

The time taken by nag_tsa_multi_inp_model_estim (g13bec) is approximately proportional to $n_{\text{xy}} \times \text{options.iter} \times n_{\text{para}}^2$.

10 Example

The data in the example relate to 40 observations of an output time series and of a single input time series. The noise series has one autoregressive (ϕ) and one seasonal moving average (Θ) argument (both of which are initially set to zero) for which the seasonal period is 4. The input series is defined by orders $b_1 = 1$, $q_1 = 0$, $p_1 = 1$, $r_1 = 3$, so that it has one ω (initially set to 2.0) and one δ (initially set to 0.5), and allows for pre-observation period effects. The constant (initially set to zero) is to be estimated so that the flag for the constant c , **options.cfixed**, remains unchanged from its default value of Nag_FALSE. Default values of **zsp** are used. Up to 20 iterations are allowed so that **options.max_iter** is set to 20, and the progress of these is monitored and solution output by setting **options.print_level** = Nag_Soln_Iter_Full. Marginal likelihood is the chosen estimation criterion so that **options.criteria** = Nag_Marginal.

After the successful call to nag_tsa_multi_inp_model_estim (g13bec), the following are computed and printed out: the correlation matrix, the residuals for the 36 differenced values and the values of z_t and n_t .

An additional example showing how to use nag_tsa_multi_inp_model_estim (g13bec) to fit a seasonal ARIMA model can be found in Byng.

10.1 Program Text

```

/* nag_tsa_multi_inp_model_estim (g13bec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <string.h>
#include <nag_string.h>
#include <nag_stdlib.h>
#include <naggl3.h>

#define XXY(I, J) xxy[(I) *tdxxy + J]

int main(void)
{
    Integer exit_status = 0;
    Integer i, inser, j, npara, nseries, nxxy, tdxxy;
    Nag_ArimaOrder arimav;
    Nag_Gl3_Opt options;
    Nag_TransfOrder transfv;
    double df, objf, *para = 0, rss, *sd = 0, *xxy = 0;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_tsa_multi_inp_model_estim (g13bec) Example Program Results\n");
#ifdef _WIN32
    scanf_s("%*[\n]"); /* Skip heading in data file */
#else
    scanf("%*[\n]"); /* Skip heading in data file */
#endif

#define CM(I, J) options.cm[(J)+(I) *options.tdcm]
#define ZT(I, J) options.zt[(J)+(I) *options.tdzt]

    /*
     * Initialize the option structure.
     */
    /* nag_tsa_options_init (g13bxc).
     * Initialization function for option setting
     */
    nag_tsa_options_init(&options);

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "", &nxxy, &nseries,
            &options.max_iter);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "", &nxxy, &nseries,
            &options.max_iter);
#endif
    if (nxxy > 0 && nseries > 0) {
        /*
         * Set some specific option variables to the desired values.
         */

        options.criteria = Nag_Marginal;
        options.print_level = Nag_Soln_Iter_Full;
        /*
         * Allocate memory to the arrays in structure transfv containing
         * the transfer function model orders of the input series.
         */
        /* nag_tsa_transf_orders (g13byc), see above. */
        nag_tsa_transf_orders(nseries, &transfv, &fail);
    }
}

```

```

/*
 * Read the orders vector of the ARIMA model for the output noise
 * component into structure arimav.
 */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT
            "" "" NAG_IFMT "" NAG_IFMT "", &arimav.p, &arimav.d, &arimav.q,
            &arimav.bigp, &arimav.bigd, &arimav.bigq, &arimav.s);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT ""
          "" NAG_IFMT "" NAG_IFMT "", &arimav.p, &arimav.d, &arimav.q,
          &arimav.bigp, &arimav.bigd, &arimav.bigq, &arimav.s);
#endif
/*
 * Read the transfer function model orders of the input series into
 * structure transfv.
 */
inser = nseries - 1;

    for (j = 0; j < inser; ++j)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &transfv.b[j]);
#else
        scanf("%" NAG_IFMT "", &transfv.b[j]);
#endif
    for (j = 0; j < inser; ++j)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &transfv.q[j]);
#else
        scanf("%" NAG_IFMT "", &transfv.q[j]);
#endif
    for (j = 0; j < inser; ++j)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &transfv.p[j]);
#else
        scanf("%" NAG_IFMT "", &transfv.p[j]);
#endif
    for (j = 0; j < inser; ++j)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &transfv.r[j]);
#else
        scanf("%" NAG_IFMT "", &transfv.r[j]);
#endif

    npara = 0;
    for (i = 0; i < inser; ++i)
        npara = npara + transfv.q[i] + transfv.p[i];
    npara = npara + arimav.p + arimav.q + arimav.bigp + arimav.bigq + nseries;
    if (npara >= 1) {
        if (!(para = NAG_ALLOC(npara, double)) ||
            !(sd = NAG_ALLOC(npara, double)) ||
            !(xxy = NAG_ALLOC(nxxy * nseries, double)))
        {
            printf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
        tdxxy = nseries;
        for (i = 0; i < npara; ++i)
#ifdef _WIN32
            scanf_s("%lf", &para[i]);
#else
            scanf("%lf", &para[i]);
#endif
        for (i = 0; i < nxxy; ++i)
            for (j = 0; j < nseries; ++j)
#ifdef _WIN32
                scanf_s("%lf", &XXY(i, j));
#else
                scanf("%lf", &XXY(i, j));
#endif
    }

```

```

/* nag_tsa_multi_inp_model_estim (g13bec), see above. */
fflush(stdout);
nag_tsa_multi_inp_model_estim(&arimav, nseries, &transfv, para,
                             npara, nxxy, xxy, tdxy, sd, &rss,
                             &objf, &df, &options, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_tsa_multi_inp_model_estim (g13bec)"
           ".\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\nThe correlation matrix is \n\n");
for (i = 0; i < npara; ++i)
    for (j = 0; j < npara; ++j)
        printf("%10.4f%c", CM(i, j), (j % 5 == 4) ? '\n' : ' ');
printf("\nThe residuals and the z and n values are\n\n");
printf("      i      res[i]      z(t)      noise(t)\n\n");
for (i = 0; i < nxxy; ++i) {
    if (i + 1 <= options.lenres) {
        printf("%4" NAG_IFMT "%15.3f", i + 1, options.res[i]);
        for (j = 0; j < nseries - 1; ++j)
            printf("%15.3f ", ZT(i, j));
        printf("%15.3f\n", options.noise[i]);
    }
}
}
else {
    printf("npara is out of range: npara = %-3" NAG_IFMT "\n", npara);
    /* nag_tsa_free (g13xzc).
     * Freeing function for use with g13 option setting
     */
    nag_tsa_free(&options);
    /* nag_tsa_trans_free (g13bzc), see above. */
    nag_tsa_trans_free(&transfv);
    exit_status = 1;
    goto END;
}
}
else {
    printf("One or both of nxxy and nseries are out of range:"
           " nxxy = %-3" NAG_IFMT " while nseries = %-3" NAG_IFMT "\n",
           nxxy, nseries);
    exit_status = 1;
    goto END;
}
/* nag_tsa_trans_free (g13bzc), see above. */
nag_tsa_trans_free(&transfv);
/* nag_tsa_free (g13xzc), see above. */
nag_tsa_free(&options);

END:
    NAG_FREE(para);
    NAG_FREE(sd);
    NAG_FREE(xxy);

    return exit_status;
}

```

10.2 Program Data

nag_tsa_multi_inp_model_estim (g13bec) Example Program Data

```

40      2      20
1      0      0      0      0      1      4
1
0
1
3
0.0      0.0      2.0      0.5      0.0

```

| | |
|-------|-------|
| 8.075 | 105.0 |
| 7.819 | 119.0 |
| 7.366 | 119.0 |
| 8.113 | 109.0 |
| 7.380 | 117.0 |
| 7.134 | 135.0 |
| 7.222 | 126.0 |
| 7.768 | 112.0 |
| 7.386 | 116.0 |
| 6.965 | 122.0 |
| 6.478 | 115.0 |
| 8.105 | 115.0 |
| 8.060 | 122.0 |
| 7.684 | 138.0 |
| 7.580 | 135.0 |
| 7.093 | 125.0 |
| 6.129 | 115.0 |
| 6.026 | 108.0 |
| 6.679 | 100.0 |
| 7.414 | 96.0 |
| 7.112 | 107.0 |
| 7.762 | 115.0 |
| 7.645 | 123.0 |
| 8.639 | 122.0 |
| 7.667 | 128.0 |
| 8.080 | 136.0 |
| 6.678 | 140.0 |
| 6.739 | 122.0 |
| 5.569 | 102.0 |
| 5.049 | 103.0 |
| 5.642 | 89.0 |
| 6.808 | 77.0 |
| 6.636 | 89.0 |
| 8.241 | 94.0 |
| 7.968 | 104.0 |
| 8.044 | 108.0 |
| 7.791 | 119.0 |
| 7.024 | 126.0 |
| 6.102 | 119.0 |
| 6.053 | 103.0 |

10.3 Program Results

nag_tsa_multi_inp_model_estim (g13bec) Example Program Results

Parameters to g13bec

```

nseries..... 2

criteria..... Nag_Marginal      cfixed..... Nag_FALSE
alpha..... 1.00e-02      beta..... 1.00e+01
delta..... 1.00e+03      gamma..... 1.00e-07
print_level... Nag_Soln_Iter_Full
outfile..... stdout

Iter = -1      Residual = 6.456655e+03      Objf = 7.097184e+03

phi           0.000000e+00
stheta        0.000000e+00
omega      series 1 2.000000e+00
delta      series 1 5.000000e-01
constant     8.688399e+01

Iter = 0      Residual = 5.802775e+03      Objf = 6.378435e+03

phi           0.000000e+00
stheta        0.000000e+00
omega      series 1 2.000000e+00
delta      series 1 5.000000e-01

```

```

constant                8.573272e+01

Iter =   1      Residual =   2.354664e+03      Objf =   2.498647e+03

phi                6.589153e-01
stheta            6.571389e-02
omega      series   1      3.721182e+00
delta      series   1      5.237968e-01
constant          5.739128e+01

Iter =   2      Residual =   1.922339e+03      Objf =   2.032375e+03

phi                6.417690e-01
stheta            -2.361191e-01
omega      series   1      4.523132e+00
delta      series   1      5.742824e-01
constant          3.814856e+01

Iter =   3      Residual =   1.530797e+03      Objf =   1.630603e+03

phi                5.550797e-01
stheta            -3.097333e-01
omega      series   1      7.697297e+00
delta      series   1      7.358370e-01
constant          -9.322197e+01

Iter =   4      Residual =   1.232926e+03      Objf =   1.324116e+03

phi                3.698329e-01
stheta            -2.145294e-01
omega      series   1      9.116523e+00
delta      series   1      6.923742e-01
constant          -9.985550e+01

Iter =   5      Residual =   1.200813e+03      Objf =   1.289272e+03

phi                3.889281e-01
stheta            -2.649652e-01
omega      series   1      8.906746e+00
delta      series   1      6.659905e-01
constant          -7.782515e+01

Iter =   6      Residual =   1.197922e+03      Objf =   1.286734e+03

phi                3.752731e-01
stheta            -2.499956e-01
omega      series   1      8.957172e+00
delta      series   1      6.616140e-01
constant          -7.656262e+01

Iter =   7      Residual =   1.197934e+03      Objf =   1.286623e+03

phi                3.804046e-01
stheta            -2.594526e-01
omega      series   1      8.954182e+00
delta      series   1      6.599012e-01
constant          -7.553429e+01

Iter =   8      Residual =   1.198009e+03      Objf =   1.286613e+03

phi                3.807082e-01
stheta            -2.567453e-01
omega      series   1      8.956063e+00
delta      series   1      6.597438e-01
constant          -7.549190e+01

Iter =   9      Residual =   1.197988e+03      Objf =   1.286612e+03

phi                3.808772e-01
stheta            -2.580559e-01
omega      series   1      8.955983e+00

```

```
delta      series  1  6.596508e-01
constant                   -7.543851e+01
```

```
Iter = 10      Residual = 1.198002e+03      Objf = 1.286611e+03
```

```
phi              3.809218e-01
stheta           -2.575832e-01
omega      series  1  8.956106e+00
delta      series  1  6.596484e-01
constant                   -7.544005e+01
```

```
Iter = 11      Residual = 1.197997e+03      Objf = 1.286611e+03
```

```
phi              3.809235e-01
stheta           -2.577863e-01
omega      series  1  8.956084e+00
delta      series  1  6.596411e-01
constant                   -7.543552e+01
```

The number of iterations carried out is 11

The final values of the parameters and their standard deviations are

| i | para[i] | sd |
|---|------------|-----------|
| 1 | 0.380924 | 0.166379 |
| 2 | -0.257786 | 0.178178 |
| 3 | 8.956084 | 0.948061 |
| 4 | 0.659641 | 0.060239 |
| 5 | -75.435521 | 33.505341 |

The residual sum of squares = 1.197997e+03

The objective function = 1.286611e+03

The degrees of freedom = 34.00

The correlation matrix is

| | | | | |
|---------|---------|---------|---------|---------|
| 1.0000 | -0.1839 | -0.1775 | -0.0340 | 0.1394 |
| -0.1839 | 1.0000 | 0.0518 | 0.2547 | -0.2860 |
| -0.1775 | 0.0518 | 1.0000 | -0.3070 | -0.2926 |
| -0.0340 | 0.2547 | -0.3070 | 1.0000 | -0.8185 |
| 0.1394 | -0.2860 | -0.2926 | -0.8185 | 1.0000 |

The residuals and the z and n values are

| i | res[i] | z(t) | noise(t) |
|----|--------|---------|----------|
| 1 | 0.397 | 180.567 | -75.567 |
| 2 | 3.086 | 191.430 | -72.430 |
| 3 | -2.818 | 196.302 | -77.302 |
| 4 | -9.941 | 195.460 | -86.460 |
| 5 | -5.061 | 201.594 | -84.594 |
| 6 | 14.053 | 199.076 | -64.076 |
| 7 | 2.624 | 195.211 | -69.211 |
| 8 | -5.823 | 193.450 | -81.450 |
| 9 | -2.147 | 197.179 | -81.179 |
| 10 | -0.216 | 196.217 | -74.217 |
| 11 | -2.517 | 191.812 | -76.812 |
| 12 | 7.916 | 184.544 | -69.544 |
| 13 | 1.423 | 194.322 | -72.322 |
| 14 | 11.936 | 200.369 | -62.369 |
| 15 | 5.117 | 200.990 | -65.990 |
| 16 | -5.672 | 200.468 | -75.468 |
| 17 | -5.681 | 195.763 | -80.763 |
| 18 | -1.637 | 184.025 | -76.025 |
| 19 | -1.019 | 175.360 | -75.360 |
| 20 | -2.623 | 175.492 | -79.492 |

| | | | |
|----|---------|---------|---------|
| 21 | 3.283 | 182.162 | -75.162 |
| 22 | 6.896 | 183.857 | -68.857 |
| 23 | 5.395 | 190.797 | -67.797 |
| 24 | 0.875 | 194.327 | -72.327 |
| 25 | -4.153 | 205.558 | -77.558 |
| 26 | 6.206 | 204.261 | -68.261 |
| 27 | 4.208 | 207.104 | -67.104 |
| 28 | -2.387 | 196.423 | -74.423 |
| 29 | -11.803 | 189.924 | -87.924 |
| 30 | 6.435 | 175.158 | -72.158 |
| 31 | 1.342 | 160.761 | -71.761 |
| 32 | -4.924 | 156.575 | -79.575 |
| 33 | 4.799 | 164.256 | -75.256 |
| 34 | -0.074 | 167.783 | -73.783 |
| 35 | -6.023 | 184.483 | -80.483 |
| 36 | -6.427 | 193.055 | -85.055 |
| 37 | -2.527 | 199.390 | -80.390 |
| 38 | 2.039 | 201.302 | -75.302 |
| 39 | 0.243 | 195.695 | -76.695 |
| 40 | -3.166 | 183.738 | -80.738 |

11 Optional Parameters

A number of optional input and output arguments to `nag_tsa_multi_inp_model_estim` (g13bec) are available through the structure argument **options** of type `Nag_G13_Opt`. An argument may be selected by assigning an appropriate value to the relevant structure member. Those arguments not selected will be assigned default values. If no use is to be made of any of the optional parameters you should use the null pointer, `G13_DEFAULT`, in place of **options** when calling `nag_tsa_multi_inp_model_estim` (g13bec); the default settings will then be used for all arguments.

Before assigning values to **options** the structure must be initialized by a call to the function `nag_tsa_options_init` (g13bxc). Values may then be assigned directly to the structure members in the normal C manner.

Options selected are checked within `nag_tsa_multi_inp_model_estim` (g13bec) for being within the required range, if outside the range, an error message is generated.

When all calls to `nag_tsa_multi_inp_model_estim` (g13bec) have been completed and the results contained in the options structure are no longer required; then `nag_tsa_free` (g13xzc) should be called to free the NAG allocated memory from **options**.

11.1 Optional Parameters Checklist and Default Values

For easy reference, the following list shows the input and output members of **options** which are valid for `nag_tsa_multi_inp_model_estim` (g13bec) together with their default values where relevant. ϵ is the *machine precision*.

| | |
|---|-------------------------------------|
| Boolean <code>options.list</code> | <code>Nag_TRUE</code> |
| <code>Nag_PrintType</code> <code>options.print_level</code> | <code>Nag_Soln</code> |
| char <code>outfile[80]</code> | <code>stdout</code> |
| void <code>(*options.print_fun)()</code> | <code>NULL</code> |
| Boolean <code>options.cfixed</code> | <code>Nag_FALSE</code> |
| <code>Nag_Likelihood</code> <code>options.criteria</code> | <code>Nag_Exact</code> |
| Integer <code>options.max_iter</code> | 50 |
| double <code>options.alpha</code> | 0.01 |
| double <code>options.beta</code> | 10.0 |
| double <code>options.delta</code> | 1000.0 |
| double <code>options.gamma</code> | $\max(100\epsilon, 10^{-7})$ |
| Integer <code>options.iter</code> | double <code>*options.cm</code> |
| double <code>*options.res</code> | Integer <code>options.lenres</code> |
| double <code>*options.zt</code> | double <code>*options.noise</code> |

11.2 Description of the Optional Parameters

list – Nag_Boolean

On entry: if **options.list** = Nag_TRUE then the argument settings which are used in the call to `nag_tsa_multi_inp_model_estim` (g13bec) will be printed.

print_level – Nag_PrintType

On entry: the level of results produced by `nag_tsa_multi_inp_model_estim` (g13bec). The following values are available:

| | |
|--------------------|---|
| Nag_NoPrint | No output. |
| Nag_Soln | The final solution. |
| Nag_Iter | One line of output for each iteration. |
| Nag_Soln_Iter | The final solution and one line of output for each iteration. |
| Nag_Soln_Iter_Full | The final solution and detailed printout at each iteration. |

Details of each level of results printout are described in Section 7.3.

Constraint: **options.print_level** = Nag_PrintNotSet, Nag_Soln, Nag_Iter, Nag_Soln_Iter or Nag_Soln_Iter_Full.

outfile – const char[80]

On entry: name of file to which the results of monitoring the course of the optimization should be printed. If **options.outfile**[0] = "\0" then the `stdout` stream is used.

print_fun – pointer to function

On entry: printing function defined by you; the prototype of **options.print_fun** is

```
void (*print_fun)(const Nag_UserPrintFun *bfx, Nag_Comm *Comm);
```

See Section 11.3.1 below for further details.

cfixed – Nag_Boolean

On entry: **options.cfixed** must be set to Nag_TRUE if the constant c is to remain fixed at its initial value, and to Nag_FALSE if it is to be estimated.

criteria – Nag_Likelihood

On entry: indicates the likelihood option for the estimation criterion. **options.criteria** must be set to Nag_LeastSquares, Nag_Exact or Nag_Marginal, to select the least squares, exact or marginal likelihood, respectively.

Constraint: **options.criteria** = Nag_LeastSquares, Nag_Exact or Nag_Marginal.

max_iter – Integer

On entry: the maximum required number of iterations. If **options.max_iter** = 0, no change is made to any of the model arguments in array **para** except that the constant c (if **options.cfixed** = Nag_FALSE) and any ω relating to simple input series are estimated. (Apart from these, estimates are always derived for the nuisance arguments relating to any backforecasts and any pre-observation period effects for transfer function inputs.)

Constraint: **options.max_iter** ≥ 0 .

alpha – double

On entry: α , the value used to constrain the magnitude of the search procedure steps (see Section 3.3).

Constraint: **options.alpha** > 0.0 .

beta – double

On entry: β , the multiplier which regulates the value of α (see Section 3.3).

Constraint: **options.beta** > 1.0.

delta – double

On entry: δ , the value of the stationarity and invertibility test tolerance factor (see Section 3.3).

Constraint: **options.delta** \geq 1.0.

gamma – double

On entry: γ , the convergence criterion (see Section 3.3).

Constraint: $0.0 \leq$ **options.gamma** < 1.0.

iter – Integer

On exit: the number of iterations carried out. A value of **options.iter** = -1 on exit indicates that the only estimates obtained up to this point have been for the nuisance arguments relating to backforecasts, unless the marginal likelihood option is used in which case estimates have also been obtained for simple input coefficients ω and for the constant c (if **options.cfixed** = Nag_FALSE). This value of **options.iter** usually indicates a failure in a consequent step of estimating transfer function input pre-observation period nuisance arguments. A value of **options.iter** = 0 on exit indicates that estimates have been obtained up to this point for the constant c (if **options.cfixed** = Nag_FALSE), for simple input coefficients ω and for the nuisance arguments relating to the backforecasts and to transfer function input pre-observation period effects.

cm – double

On exit: this pointer is allocated memory internally with **npara** \times **npara** elements corresponding to **npara** rows by **npara** columns. The **npara** rows and columns of **options.cm** contain the correlation coefficients relating to each pair of arguments in **para**. All coefficients relating to the constant will be zero if the constant is fixed. However, if the function fails to invert the second derivative matrix, in which case **fail** will have an exit value of NE_MAT_NOT_POS_DEF, then the contents of **options.cm** will be indeterminate.

res – double

On exit: the values of the residuals relating to the differenced values of the output series. This pointer is allocated memory internally with **options.lenres** elements.

lenres – Integer

On exit: the length of **options.res**.

zt – double

On exit: this pointer is allocated memory internally with **nxy** \times (**nseries** - 1) elements corresponding to **nxy** rows by (**nseries** - 1) columns. The columns of **options.zt** hold the values of the input component series z_t .

noise – double

On exit: this pointer is allocated memory internally with **nxy** elements. It holds the output noise component n_t .

11.3 Description of Printed Output

The level of printed output can be controlled with the structure members **options.list** and **options.print_level**, see section 7.2. If **options.list** = Nag_TRUE then the argument values to `nag_tsa_multi_inp_model_estim` (g13bec) are listed, whereas the printout of results is governed by the value of **options.print_level**. The default of **options.print_level** = Nag_Soln which provides a

printout of the final solution. This section describes all of the possible levels of results printout available from `nag_tsa_multi_inp_model_estim` (g13bec). When **options.print_level** = `Nag_Iter` or `Nag_Soln_Iter` a single line of output is produced at each iteration, this gives the following values.

| | |
|----------|--|
| Iter | the current iteration number, options.iter . |
| Residual | the residual sum of squares, print_fun → rss . |
| Objf | the objective function at the latest set of parameter estimates. |

When **options.print_level** = `Nag_Soln_Iter_Full` a description and value for each of the arguments in the `para` array is output. The descriptions are ϕ for ϕ , θ for θ , Φ for Φ , Θ for Θ , ω for ω , δ for δ and c for c . In a simple input, ω for ω in a transfer function input, **options.delta** for δ and constant for c . In addition series 1, series 2, etc, indicate the input series relevant to the omega and **options.delta** arguments.

If **options.print_level** = `Nag_Soln`, `Nag_Soln_Iter` or `Nag_Soln_Iter_Full` the final solution is printed out. This consists of:

| | |
|--------------|---------------------------------------|
| i | the argument number. |
| para[i] | the values of the argument. |
| sd | the standard deviations. |
| options.iter | the number of iterations carried out. |
| rss | the residual sum of squares. |
| objf | the objective function. |
| df | the degrees of freedom. |

If **options.print_level** = `Nag_NoPrint` then printout will be suppressed; you can print the final solution when `nag_tsa_multi_inp_model_estim` (g13bec) returns to the calling program.

11.3.1 Output of results via a user-defined printing function

You may also specify their own print function for output of iteration results and the final solution by use of the **options.print_fun** function pointer, prototype

```
void (*print_fun) (const Nag_UserPrintFun *bfx, Nag_Comm *Comm);
```

The rest of this section can be skipped if the default printing facilities provide the required functionality. When a user-defined function is assigned to **options.print_fun** this will be called in preference to the internal print function of `nag_tsa_multi_inp_model_estim` (g13bec). Calls to the user-defined function are again controlled by means of the **options.print_level** member. Information is provided through two structure arguments to **options.print_fun**, the structure of type `Nag_UserPrintFun` contains the following members relevant to `nag_tsa_multi_inp_model_estim` (g13bec):

itc – Integer

The number of the particular iteration being monitored.

rss – double *

The residual sum of squares, S , at the latest set of valid parameter estimates.

objf – double *

The objective function, D , at the latest set of valid parameter estimates.

para – double *

The pointer to memory containing **print_fun**→**npara** latest values of the estimates of the multi-input model arguments.

npara – Integer

The exact number of ϕ , θ , Φ , Θ , ω , δ and c arguments.

npe – Integer

The number of ARIMA (ϕ , θ , Φ , Θ), omega (ω), **options.delta** (δ), and c arguments being estimated.

mtyp – Integer

mser – Integer

The pointers to memory, each with **print_fun**→**npe** elements. The value of each element in **print_fun**→**mtyp** and **print_fun**→**mser** corresponds to the description of each parameter estimated in **print_fun**→**para**. The following should be read in conjunction with the description of the argument **print**. The relevant description for the value of **print_fun**→**para** is:

| print_fun → mtyp [i] | Description | |
|--|----------------------|---|
| 1 | phi | |
| 2 | theta | |
| 3 | sphi | |
| 4 | stheta | |
| 5 | omega/si | series print_fun → mser [i] |
| 6 | omega | series print_fun → mser [i] |
| 7 | options.delta | series print_fun → mser [i] |
| 8 | constant | |

for $i = 0, 1, \dots, \mathbf{print_fun} \rightarrow \mathbf{npe}$. For the phi, theta, sphi, stheta and constant arguments, **print_fun**→**mser**[i] = 0.

sd – double *

The pointer to memory containing the npara values of the standard deviations.

df – double *

The number of degrees of freedom associated with S .
