

# NAG Library Function Document

## nag\_tsa\_diff (g13aac)

### 1 Purpose

nag\_tsa\_diff (g13aac) carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

### 2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_diff (const double x[], Integer nx, Integer d, Integer ds,
                  Integer s, double xd[], Integer *nxd, NagError *fail)
```

### 3 Description

Let  $\nabla^d \nabla_s^D x_i$  be the  $i$ th value of a time series  $x_i$ , for  $i = 1, 2, \dots, n$  after non-seasonal differencing of order  $d$  and seasonal differencing of order  $D$  (with period or seasonality  $s$ ). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i & d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i & D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order  $d$  is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i & \text{for } i = 1, 2, \dots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i & \text{for } i = 1, 2, \dots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i & \text{for } i = 1, 2, \dots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order  $D$  is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i & \text{for } i = 1, 2, \dots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i & \text{for } i = 1, 2, \dots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i & \text{for } i = 1, 2, \dots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of  $m = n - d - D \times s$  values.

### 4 References

None.

### 5 Arguments

- 1: **x[nx]** – const double *Input*  
*On entry:* the undifferenced time series,  $x_i$ , for  $i = 1, 2, \dots, n$ .

- 2: **nx** – Integer *Input*  
*On entry:*  $n$ , the number of values in the undifferenced time series.  
*Constraint:*  $\mathbf{nx} > \mathbf{d} + (\mathbf{ds} \times \mathbf{s})$ .
- 3: **d** – Integer *Input*  
*On entry:*  $d$ , the order of non-seasonal differencing.  
*Constraint:*  $\mathbf{d} \geq 0$ .
- 4: **ds** – Integer *Input*  
*On entry:*  $D$ , the order of seasonal differencing.  
*Constraint:*  $\mathbf{ds} \geq 0$ .
- 5: **s** – Integer *Input*  
*On entry:*  $s$ , the seasonality.  
*Constraints:*  
     if  $\mathbf{ds} > 0$ ,  $\mathbf{s} > 0$ ;  
     if  $\mathbf{ds} = 0$ ,  $\mathbf{s} \geq 0$ .
- 6: **xd[nx]** – double *Output*  
*On exit:* the differenced values in elements 0 to  $\mathbf{nxd} - 1$ , and reconstitution data in the remainder of the array.
- 7: **nxd** – Integer \* *Output*  
*On exit:* the number of differenced values in the array **xd**.
- 8: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{d} = \langle value \rangle$ .

Constraint:  $\mathbf{d} \geq 0$ .

On entry,  $\mathbf{ds} = \langle value \rangle$ .

Constraint:  $\mathbf{ds} \geq 0$ .

On entry,  $\mathbf{s} = \langle value \rangle$ .

Constraint:  $\mathbf{s} \geq 0$ .

**NE\_INT\_2**

On entry, **ds** =  $\langle value \rangle$ .  
 Constraint: if **s** = 0, **ds**  $\leq$  0.

**NE\_INT\_4**

On entry, **nx** =  $\langle value \rangle$ , **d** =  $\langle value \rangle$ , **ds** =  $\langle value \rangle$  and **s** =  $\langle value \rangle$ .  
 Constraint: **nx**  $>$  **d** + (**ds**  $\times$  **s**).

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

The computations are believed to be stable.

**8 Parallelism and Performance**

nag\_tsa\_diff (g13aac) is not threaded in any implementation.

**9 Further Comments**

The time taken by nag\_tsa\_diff (g13aac) is approximately proportional to  $(\mathbf{d} + \mathbf{ds}) \times \mathbf{nx}$ .

**10 Example**

This example reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

**10.1 Program Text**

```
/* nag_tsa_diff (g13aac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    Integer exit_status, i, d, ds, s, nx, nxd;
    NagError fail;
```

```

/* Arrays */
double *x = 0, *xd = 0;

INIT_FAIL(fail);

exit_status = 0;
printf("nag_tsa_diff (g13aac) Example Program Results\n");

/* Skip heading in data file */
#ifdef _WIN32
scanf_s("%*[\n] ");
#else
scanf("%*[\n] ");
#endif

#ifdef _WIN32
scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nx,
        &d, &ds, &s);
#else
scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nx,
        &d, &ds, &s);
#endif

if (nx > 0) {
    /* Allocate memory */
    if (!(x = NAG_ALLOC(nx, double)) || !(xd = NAG_ALLOC(nx, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    for (i = 1; i <= nx; ++i)
#ifdef _WIN32
        scanf_s("%lf", &x[i - 1]);
#else
        scanf("%lf", &x[i - 1]);
#endif
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif

    printf("\n");
    printf("Non-seasonal differencing of order %" NAG_IFMT " "
           "and seasonal differencing\nof order %" NAG_IFMT " "
           "with seasonality %" NAG_IFMT " are applied\n", d, ds, s);

    /* nag_tsa_diff (g13aac).
     * Univariate time series, seasonal and non-seasonal
     * differencing
     */
    nag_tsa_diff(x, nx, d, ds, s, xd, &nxd, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_diff (g13aac).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("\n");
    printf("The output array holds %2" NAG_IFMT " values, of which the "
           "first %2" NAG_IFMT " are differenced values\n\n", nx, nxd);

    for (i = 1; i <= nx; ++i) {
        printf("%10.1f", xd[i - 1]);
        if (i % 5 == 0 || i == nx)
            printf("\n");
    }
}

```

```

END:
    NAG_FREE(x);
    NAG_FREE(xd);

    return exit_status;
}

```

## 10.2 Program Data

```

nag_tsa_diff (g13aac) Example Program Data
20 2 1 4
120.0 108.0 98.0 118.0 135.0
131.0 118.0 125.0 121.0 100.0
82.0 82.0 89.0 88.0 86.0
96.0 108.0 110.0 99.0 105.0

```

## 10.3 Program Results

```

nag_tsa_diff (g13aac) Example Program Results

```

Non-seasonal differencing of order 2 and seasonal differencing  
of order 1 with seasonality 4 are applied

The output array holds 20 values, of which the first 14 are differenced values

-11.0	-10.0	-8.0	4.0	12.0
-2.0	18.0	9.0	-4.0	-6.0
-5.0	-2.0	-12.0	5.0	2.0
-10.0	-13.0	17.0	6.0	105.0

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