

# NAG Library Function Document

## nag\_wilcoxon\_test (g08agc)

### 1 Purpose

nag\_wilcoxon\_test (g08agc) performs the Wilcoxon signed rank test on a single sample of size  $n$ .

### 2 Specification

```
#include <nag.h>
#include <nagg08.h>

void nag_wilcoxon_test (Integer n, const double x[], double median,
    Nag_TailProbability tail, Nag_IncSignZeros zeros, double *w, double *z,
    double *p, Integer *non_zero, NagError *fail)
```

### 3 Description

The Wilcoxon one sample signed rank test may be used to test whether a particular sample came from a population with a specified median. It is assumed that the population distribution is symmetric. The data consist of a single sample of  $n$  observations denoted by  $x_1, x_2, \dots, x_n$ . This sample may arise from the difference between pairs of observations from two matched samples of equal size taken from two populations, in which case the test may be used to test whether the median of the first population is the same as that of the second population.

The hypothesis under test,  $H_0$ , often called the null hypothesis, is that the median is equal to some given value ( $X_{med}$ ), and this is to be tested against an alternative hypothesis  $H_1$  which is

$H_1$  : population median  $\neq X_{med}$ ; or

$H_1$  : population median  $> X_{med}$ ; or

$H_1$  : population median  $< X_{med}$ ,

using a two tailed, upper tailed or lower tailed probability respectively. You select the alternative hypothesis by choosing the appropriate tail probability to be computed (see the description of argument **tail** in Section 5).

The Wilcoxon test differs from the Sign test (see nag\_sign\_test (g08aac)) in that the magnitude of the scores is taken into account, rather than simply the direction of such scores.

The test procedure is as follows:

- (a) For each  $x_i$ , for  $i = 1, 2, \dots, n$ , the signed difference  $d_i = x_i - X_{med}$  is found, where  $X_{med}$  is a given test value for the median of the sample.
- (b) The absolute differences  $|d_i|$  are ranked with rank  $r_i$  and any tied values of  $|d_i|$  are assigned the average of the tied ranks. You may choose whether or not to ignore any cases where  $d_i = 0$  by removing them before or after ranking (see the description of the argument **zeros** in Section 5).
- (c) The number of nonzero  $d_i$ 's is found.
- (d) To each rank is affixed the sign of the  $d_i$  to which it corresponds. Let  $s_i = \text{sign}(d_i)r_i$ .
- (e) The sum of the positive-signed ranks,  $W = \sum_{s_i > 0} s_i = \sum_{i=1}^n \max(s_i, 0.0)$ , is calculated.

nag\_wilcoxon\_test (g08agc) returns:

- (a) The test statistic  $W$ ;
- (b) The number  $n_1$  of nonzero  $d_i$ 's;

(c) The approximate Normal test statistic  $z$ , where

$$z = \frac{\left(W - \frac{n_1(n_1+1)}{4}\right) - \text{sign}\left(W - \frac{n_1(n_1+1)}{4}\right) \times \frac{1}{2}}{\sqrt{\frac{1}{4} \sum_{i=1}^n s_i^2}}$$

(d) The tail probability,  $p$ , corresponding to  $W$ , depending on the choice of the alternative hypothesis,  $H_1$ .

If  $n_1 \leq 80$ ,  $p$  is computed exactly; otherwise, an approximation to  $p$  is returned based on an approximate Normal statistic corrected for continuity according to the tail specified.

The value of  $p$  can be used to perform a significance test on the median against the alternative hypothesis. Let  $\alpha$  be the size of the significance test (that is,  $\alpha$  is the probability of rejecting  $H_0$  when  $H_0$  is true). If  $p < \alpha$  then the null hypothesis is rejected. Typically  $\alpha$  might be 0.05 or 0.01.

## 4 References

Conover W J (1980) *Practical Nonparametric Statistics* Wiley

Neumann N (1988) Some procedures for calculating the distributions of elementary nonparametric test statistics *Statistical Software Newsletter* **14(3)** 120–126

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

## 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:* the size of the sample,  $n$ .  
*Constraint:*  $n \geq 1$ .
- 2: **x[n]** – const double *Input*  
*On entry:* the sample observations,  $x_1, x_2, \dots, x_n$ .
- 3: **median** – double *Input*  
*On entry:* the median test value,  $X_{med}$ .
- 4: **tail** – Nag\_TailProbability *Input*  
*On entry:* indicates the choice of tail probability, and hence the alternative hypothesis.  
**tail** = Nag\_TwoTail  
A two tailed probability is calculated and the alternative hypothesis is  $H_1$ : population median  $\neq X_{med}$ .  
**tail** = Nag\_UpperTail  
An upper tailed probability is calculated and the alternative hypothesis is  $H_1$ : population median  $> X_{med}$ .  
**tail** = Nag\_LowerTail  
A lower tailed probability is calculated and the alternative hypothesis is  $H_1$ : population median  $< X_{med}$ .  
*Constraint:* **tail** = Nag\_TwoTail, Nag\_UpperTail or Nag\_LowerTail.
- 5: **zeros** – Nag\_IncSignZeros *Input*  
*On entry:* indicates whether or not to include the cases where  $d_i = 0.0$  in the ranking of the  $d_i$ 's.  
**zeros** = Nag\_IncSignZerosY  
All  $d_i = 0.0$  are included when ranking.

**zeros** = Nag\_IncSignZerosN

All  $d_i = 0.0$ , are ignored, that is all cases where  $d_i = 0.0$  are removed before ranking.

*Constraint:* **zeros** = Nag\_IncSignZerosY or Nag\_IncSignZerosN.

- 6:   **w** – double \* *Output*  
       *On exit:* the Wilcoxon rank sum statistic,  $W$ , being the sum of the positive ranks.
- 7:   **z** – double \* *Output*  
       *On exit:* the approximate Normal test statistic,  $z$ , as described in Section 3.
- 8:   **p** – double \* *Output*  
       *On exit:* the tail probability,  $p$ , as specified by the argument **tail**.
- 9:   **non\_zero** – Integer \* *Output*  
       *On exit:* the number of nonzero  $d_i$ 's,  $n_1$ .
- 10:   **fail** – NagError \* *Input/Output*  
       The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument **tail** had an illegal value.

On entry, argument **zeros** had an illegal value.

### NE\_G08AG\_SAMP\_IDEN

The whole sample is identical to the given median test value.

### NE\_INT\_ARG\_LT

On entry, **n** =  $\langle value \rangle$ .

*Constraint:* **n**  $\geq 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

## 7 Accuracy

The approximation used to calculate  $p$  when  $n_1 > 80$  will return a value with a relative error of less than 10 percent for most cases. The error may increase for cases where there are a large number of ties in the sample.

## 8 Parallelism and Performance

nag\_wilcoxon\_test (g08agc) is not threaded in any implementation.

## 9 Further Comments

The time taken by `nag_wilcoxon_test` (g08agc) increases with  $n_1$ , until  $n_1 > 80$ , from which point on the approximation is used. The time decreases significantly at this point and increases again modestly with  $n_1$  for  $n_1 > 80$ .

## 10 Example

The example program performs the Wilcoxon signed rank test on two matched samples of size 8, taken from two populations. The distribution of the differences between pairs of observations from the two populations is assumed to be symmetric. The test is used to test whether the medians of the two distributions of the populations are equal or not. The test statistic, the approximate Normal statistic and the two tailed probability are computed and printed.

### 10.1 Program Text

```
/* nag_wilcoxon_test (g08agc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg08.h>

int main(void)
{
    Integer exit_status = 0, i, n, non_zero;
    NagError fail;
    double *data = 0, median, p, w, *x = 0, *y = 0, z;

    INIT_FAIL(fail);

    printf("nag_wilcoxon_test (g08agc) Example Program Results");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[^\\n]");
#else
    scanf("%*[^\\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &n);
#else
    scanf("%" NAG_IFMT " ", &n);
#endif
    if (!(x = NAG_ALLOC(n, double))
        || !(y = NAG_ALLOC(n, double))
        || !(data = NAG_ALLOC(n, double)))
    {
        printf("Allocation failure\\n");
        exit_status = -1;
        goto END;
    }

    for (i = 1; i <= n; ++i)
#ifdef _WIN32
        scanf_s("%lf", &x[i - 1]);
#else
        scanf("%lf", &x[i - 1]);
#endif
    #endif
```

```

    for (i = 1; i <= n; ++i)
#ifdef _WIN32
        scanf_s("%lf", &y[i - 1]);
#else
        scanf("%lf", &y[i - 1]);
#endif

    printf("\n\n");
    printf("%s\n", "Wilcoxon one sample signed ranks test");
    printf("\n");
    printf("%s", "Data values\n");
    for (i = 1; i <= n; ++i)
        printf("%5.1f%s", x[i - 1], i % 8 ? "" : "\n");
    for (i = 1; i <= n; ++i)
        printf("%5.1f%s", y[i - 1], i % 8 ? "" : "\n");

    for (i = 1; i <= n; ++i)
        data[i - 1] = x[i - 1] - y[i - 1];
    median = 0.0;
    /* nag_wilcoxon_test (g08agc).
     * Performs the Wilcoxon one-sample (matched pairs) signed
     * rank test
     */
    nag_wilcoxon_test(n, data, median, Nag_TwoTail, Nag_IncSignZerosN, &w, &z,
                     &p, &non_zero, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_wilcoxon_test (g08agc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("\n\n");
    printf("%s%8.4f\n", "Test statistic", w);
    printf("%s%8.4f\n", "Normalized test statistic", z);
    printf("%s%8 NAG_IFMT\n", "Degrees of freedom", non_zero);
    printf("%s%8.4f\n", "Two tail probability", p);

END:
    NAG_FREE(x);
    NAG_FREE(y);
    NAG_FREE(data);
    return exit_status;
}

```

## 10.2 Program Data

nag\_wilcoxon\_test (g08agc) Example Program Data

```

8
82 69 73 43 58 56 76 65
63 42 74 37 51 43 80 62

```

## 10.3 Program Results

nag\_wilcoxon\_test (g08agc) Example Program Results

Wilcoxon one sample signed ranks test

Data values

```

82.0 69.0 73.0 43.0 58.0 56.0 76.0 65.0
63.0 42.0 74.0 37.0 51.0 43.0 80.0 62.0

```

```

Test statistic           = 32.0000
Normalized test statistic = 1.8904
Degrees of freedom       = 8
Two tail probability      = 0.0547

```

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