NAG Library Function Document

nag_rand_gen_multinomial (g05tgc)

1 Purpose

nag_rand_gen_multinomial (g05tgc) generates a sequence of n variates, each consisting of k pseudorandom integers, from the discrete multinomial distribution with k outcomes and m trials, where the outcomes have probabilities $p_1, p_2, \ldots, p_k$ respectively.

2 Specification

```
#include <nag.h>
#include <nagg05.h>
void nag_rand_gen_multinomial (Nag_OrderType order, Nag_ModeRNG mode,
     Integer n, Integer m, Integer k, const double p[], double r[],
     Integer lr, Integer state[], Integer x[], Integer pdx, NagError *fail)
```

3 Description

nag_rand_gen_multinomial (g05tgc) generates a sequence of n groups of k integers $x_{i,j}$, for $j = 1, 2, \ldots, k$ and $i = 1, 2, \ldots, n$, from a multinomial distribution with m trials and k outcomes, where the probability of $x_{i,j} = I_j$ for each $j = 1, 2, \ldots, k$ is

$$P(i_1 = I_1, \ldots, i_k = I_k) = \frac{m!}{\prod_{j=1}^{k} I_j!} \prod_{j=1}^{k} p_j^{I_j} = \frac{m!}{I_1! I_2! \cdots I_k!} p_1^{I_1} p_2^{I_2} \cdots p_k^{I_k},$$

where

$$\sum_{j=1}^{k} p_j = 1 \quad \text{and} \quad \sum_{j=1}^{k} I_j = m.$$

A single trial can have several outcomes (k) and the probability of achieving each outcome is known (p_j). After m trials each outcome will have occurred a certain number of times. The k numbers representing the numbers of occurrences for each outcome after m trials is then a single sample from the multinomial distribution defined by the parameters k, m and p_j, for $j = 1, 2, \ldots, k$. This function returns n such samples.

When $k = 2$ this distribution is equivalent to the binomial distribution with parameters m and $p = p_1$ (see nag_rand_binomial (g05tac)).

The variates can be generated with or without using a search table and index. If a search table is used then it is stored with the index in a reference vector and subsequent calls to nag_rand_gen_multinomial (g05tgc) with the same parameter values can then use this reference vector to generate further variates. The reference array is generated only for the outcome with greatest probability. The number of successes for the outcome with greatest probability is calculated first as for the binomial distribution (see nag_rand_binomial (g05tac)); the number of successes for other outcomes are calculated in turn for the remaining reduced multinomial distribution; the number of successes for the final outcome is simply calculated to ensure that the total number of successes is m.

One of the initialization functions nag_rand_init_repeatable (g05kfc) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeatable (g05kgc) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_gen_multinomial (g05tgc).

4 References

Knuth D E (1981) *The Art of Computer Programming (Volume 2)* (2nd Edition) Addison–Wesley

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **mode** – Nag_ModeRNG *Input*

On entry: a code for selecting the operation to be performed by the function.

mode = Nag_InitializeReference
Set up reference vector only.

mode = Nag_GenerateFromReference
Generate variates using reference vector set up in a prior call to nag_rand_gen_multinomial (g05tgc).

mode = Nag_InitializeAndGenerate
Set up reference vector and generate variates.

mode = Nag_GenerateWithoutReference
Generate variates without using the reference vector.

Constraint: **mode** = Nag_InitializeReference, Nag_GenerateFromReference, Nag_InitializeAndGenerate or Nag_GenerateWithoutReference.

3: **n** – Integer *Input*

On entry: n, the number of pseudorandom numbers to be generated.

Constraint: $\mathbf{n} \geq 0$.

4: **m** – Integer *Input*

On entry: m, the number of trials of the multinomial distribution.

Constraint: $\mathbf{m} \geq 0$.

5: **k** – Integer *Input*

On entry: k, the number of possible outcomes of the multinomial distribution.

Constraint: $\mathbf{k} \geq 2$.

6: **p**[**k**] – const double *Input*

On entry: contains the probabilities p_j, for $j = 1, 2, \ldots, k$, of the k possible outcomes of the multinomial distribution.

Constraint: $0.0 \leq \mathbf{p}[j-1] \leq 1.0$ and $\sum_{j=1}^{k} \mathbf{p}[j-1] = 1.0$.

7: **r**[**lr**] – double *Communication Array*

On entry: if **mode** = Nag_GenerateFromReference, the reference vector from the previous call to nag_rand_gen_multinomial (g05tgc).

If **mode** = Nag_GenerateWithoutReference, **r** is not referenced and may be **NULL**.

On exit: if **mode** ≠ Nag_GenerateWithoutReference, the reference vector.

8: **lr** – Integer *Input*

Note: for convenience p_max will be used here to denote the expression $p_max = \max_j(\mathbf{p}[j])$.

On entry: the dimension of the array **r**.

Suggested value:

if **mode** ≠ Nag_GenerateWithoutReference, $\mathbf{lr} = 30 + 20 \times \sqrt{\mathbf{m} \times p_max \times (1 - p_max)}$; otherwise $\mathbf{lr} = 1$.

Constraints:

if **mode** = Nag_InitializeReference or Nag_InitializeAndGenerate,

$$\begin{aligned}\mathbf{lr} \quad > \quad & \min\left(\mathbf{m}, \text{INT}\left[\mathbf{m} \times p_max + 7.25 \times \sqrt{\mathbf{m} \times p_max \times (1 - p_max)} + 8.5\right]\right) \\ & - \max\left(0, \text{INT}\left[\mathbf{m} \times p_max - 7.25 \times \sqrt{\mathbf{m} \times p_max \times (1 - p_max)}\right]\right) + 9\end{aligned};$$

if **mode** = Nag_GenerateFromReference, **lr** must remain unchanged from the previous call to nag_rand_gen_multinomial (g05tgc).

9: **state**[dim] – Integer *Communication Array*

Note: the dimension, dim, of this array is dictated by the requirements of associated functions that must have been previously called. This array MUST be the same array passed as argument **state** in the previous call to nag_rand_init_repeatable (g05kfc) or nag_rand_init_nonrepeatable (g05kgc).

On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

10: **x**[dim] – Integer *Output*

Note: the dimension, *dim*, of the array **x** must be at least

$\max(1, \mathbf{pdx} \times \mathbf{k})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{n} \times \mathbf{pdx})$ when **order** = Nag_RowMajor.

Where $\mathbf{X}(i, j)$ appears in this document, it refers to the array element

$\mathbf{x}[(j-1) \times \mathbf{pdx} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{x}[(i-1) \times \mathbf{pdx} + j - 1]$ when **order** = Nag_RowMajor.

On exit: the first n rows of $\mathbf{X}(i, j)$ each contain k pseudorandom numbers representing a k-dimensional variate from the specified multinomial distribution.

11: **pdx** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **x**.

Constraints:

if **order** = Nag_ColMajor, $\mathbf{pdx} \geq \mathbf{n}$;
if **order** = Nag_RowMajor, $\mathbf{pdx} \geq \mathbf{k}$.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **k** = ⟨*value*⟩.
Constraint: **k** ≥ 2.

On entry, **lr** is too small when **mode** = Nag_InitializeReference or Nag_InitializeAndGenerate: **lr** = ⟨*value*⟩, minimum length required = ⟨*value*⟩.

On entry, **m** = ⟨*value*⟩.
Constraint: **m** ≥ 0.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_INT_2

On entry, **pdx** = ⟨*value*⟩ and **k** = ⟨*value*⟩.
Constraint: **pdx** ≥ **k**.

On entry, **pdx** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: **pdx** ≥ **n**.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_INVALID_STATE

On entry, **state** vector has been corrupted or not initialized.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_PREV_CALL

The value of **m** or **k** is not the same as when **r** was set up in a previous call.
Previous value of **m** = ⟨*value*⟩ and **m** = ⟨*value*⟩.
Previous value of **k** = ⟨*value*⟩ and **k** = ⟨*value*⟩.

NE_REAL_ARRAY

On entry, at least one element of the vector **p** is less than 0.0 or greater than 1.0.

On entry, the sum of the elements of **p** do not equal one.

NE_REF_VEC

On entry, some of the elements of the array **r** have been corrupted or have not been initialized.

7 Accuracy

Not applicable.

8 Parallelism and Performance

nag_rand_gen_multinomial (g05tgc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The reference vector for only one outcome can be set up because the conditional distributions cannot be known in advance of the generation of variates. The outcome with greatest probability of success is chosen for the reference vector because it will have the greatest spread of likely values.

10 Example

This example prints 20 pseudorandom k-dimensional variates from a multinomial distribution with $k = 4$, $m = 6000$, $p_1 = 0.08$, $p_2 = 0.1$, $p_3 = 0.8$ and $p_4 = 0.02$, generated by a single call to nag_rand_gen_multinomial (g05tgc), after initialization by nag_rand_init_repeatable (g05kfc).

10.1 Program Text

```
/* nag_rand_gen_multinomial (g05tgc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg05.h>

#define X(I, J) x[(order == Nag_ColMajor)?(J*pdx + I):(I*pdx + J)]

int main(void)
{
  /* Integer scalar and array declarations */
  Integer exit_status = 0;
  Integer lr, x_size, i, j, lstate, pdx;
  Integer *state = 0, *x = 0;

  /* NAG structures */
  NagError fail;
  Nag_ModeRNG mode;

  /* Double scalar and array declarations */
  double p_max;
  double *r = 0;

  /* Set the distribution parameters */
  Integer k = 4;
  Integer m = 6000;
  double p[] = { 0.08e0, 0.1e0, 0.8e0, 0.02e0 };

  /* Set the sample size */
  Integer n = 20;

  /* Return the results in column major order */
  Nag_OrderType order = Nag_ColMajor;
```

```
  /* Choose the base generator */
  Nag_BaseRNG genid = Nag_Basic;
  Integer subid = 0;

  /* Set the seed */
  Integer seed[] = { 1762543 };
  Integer lseed = 1;

  /* Initialize the error structure */
  INIT_FAIL(fail);

  printf("nag_rand_gen_multinomial (g05tgc) Example Program Results\n\n");

  /* Get the length of the state array */
  lstate = -1;
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  pdx = (order == Nag_ColMajor) ? n : k;
  x_size = (order == Nag_ColMajor) ? pdx * k : pdx * n;

  /* Calculate the size of the reference vector */
  p_max = 0.0;
  for (i = 1; i < k; i++)
    p_max = (p_max < p[i]) ? p[i] : p_max;
  lr = 30 + 20 * sqrt(m * p_max * (1 - p_max));

  /* Allocate arrays */
  if (!(r = NAG_ALLOC(lr, double)) ||
      !(state = NAG_ALLOC(lstate, Integer)) ||
      !(x = NAG_ALLOC(x_size, Integer)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Initialize the generator to a repeatable sequence */
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  /* Generate the variates, initializing the reference vector
     at the same time */
  mode = Nag_InitializeAndGenerate;
  nag_rand_gen_multinomial(order, mode, n, m, k, p, r, lr, state, x, pdx,
                           &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_gen_multinomial (g05tgc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  /* Display the variates */
  for (i = 0; i < n; i++) {
    for (j = 0; j < k; j++)
      printf("%12" NAG_IFMT "", X(i, j));
    printf("\n");
  }

END:
  NAG_FREE(r);
  NAG_FREE(state);
  NAG_FREE(x);

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_rand_gen_multinomial (g05tgc) Example Program Results

         468         603        4811         118
         490         630        4761         119
         482         575        4821         122
         495         591        4826          88
         512         611        4761         116
         474         601        4800         125
         485         595        4791         129
         468         582        4825         125
         485         598        4800         117
         485         573        4814         128
         501         634        4749         116
         482         618        4780         120
         470         584        4810         136
         479         642        4750         129
         476         608        4807         109
         473         631        4782         114
         509         596        4778         117
         450         565        4877         108
         484         556        4840         120
         466         615        4802         117
```