

# NAG Library Function Document

## nag\_robust\_m\_regsn\_wts (g02hbc)

### 1 Purpose

nag\_robust\_m\_regsn\_wts (g02hbc) finds, for a real matrix  $X$  of full column rank, a lower triangular matrix  $A$  such that  $(A^T A)^{-1}$  is proportional to a robust estimate of the covariance of the variables. nag\_robust\_m\_regsn\_wts (g02hbc) is intended for the calculation of weights of bounded influence regression using nag\_robust\_m\_regsn\_user\_fn (g02hdc).

### 2 Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_robust_m_regsn_wts (Nag_OrderType order,
    double (*ucv)(double t, Nag_Comm *comm),
    Integer n, Integer m, const double x[], Integer pdx, double a[],
    double z[], double bl, double bd, double tol, Integer maxit,
    Integer nitmon, const char *outfile, Integer *nit, Nag_Comm *comm,
    NagError *fail)
```

### 3 Description

In fitting the linear regression model

$$y = X\theta + \epsilon,$$

where  $y$  is a vector of length  $n$  of the dependent variable,

$X$  is an  $n$  by  $m$  matrix of independent variables,

$\theta$  is a vector of length  $m$  of unknown arguments,

and  $\epsilon$  is a vector of length  $n$  of unknown errors,

it may be desirable to bound the influence of rows of the  $X$  matrix. This can be achieved by calculating a weight for each observation. Several schemes for calculating weights have been proposed (see Hampel *et al.* (1986) and Marazzi (1987)). As the different independent variables may be measured on different scales one group of proposed weights aims to bound a standardized measure of influence. To obtain such weights the matrix  $A$  has to be found such that

$$\frac{1}{n} \sum_{i=1}^n u(\|z_i\|_2) z_i z_i^T = I \quad (I \text{ is the identity matrix})$$

and

$$z_i = Ax_i,$$

where  $x_i$  is a vector of length  $m$  containing the elements of the  $i$ th row of  $X$ ,

$A$  is an  $m$  by  $m$  lower triangular matrix,

$z_i$  is a vector of length  $m$ ,

and  $u$  is a suitable function.

The weights for use with nag\_robust\_m\_regsn\_user\_fn (g02hdc) may then be computed using

$$w_i = f(\|z_i\|_2)$$

for a suitable user-supplied function  $f$ .

nag\_robust\_m\_regsn\_wts (g02hbc) finds  $A$  using the iterative procedure

$$A_k = (S_k + I)A_{k-1},$$

where  $S_k = (s_{jl})$ , for  $j = 1, 2, \dots, m$  and  $l = 1, 2, \dots, m$ , is a lower triangular matrix such that

$$s_{jl} = \begin{cases} -\min[\max(h_{jl}/n, -BL), BL], & j > l \\ -\min[\max(\frac{1}{2}(h_{jj}/n - 1), -BD), BD], & j = l \end{cases}$$

$$h_{jl} = \sum_{i=1}^n u(\|z_i\|_2) z_{ij} z_{il}$$

and  $BD$  and  $BL$  are suitable bounds.

In addition the values of  $\|z_i\|_2$ , for  $i = 1, 2, \dots, n$ , are calculated.

nag\_robust\_m\_regsn\_wts (g02hbc) is based on routines in ROBETH; see Marazzi (1987).

## 4 References

Hampel F R, Ronchetti E M, Rousseeuw P J and Stahel W A (1986) *Robust Statistics. The Approach Based on Influence Functions* Wiley

Huber P J (1981) *Robust Statistics* Wiley

Marazzi A (1987) Weights for bounded influence regression in ROBETH *Cah. Rech. Doc. IUMSP, No. 3 ROB 3* Institut Universitaire de Médecine Sociale et Préventive, Lausanne

## 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **ucv** – function, supplied by the user *External Function*

**ucv** must return the value of the function  $u$  for a given value of its argument. The value of  $u$  must be non-negative.

The specification of **ucv** is:

```
double ucv (double t, Nag_Comm *comm)
```

1: **t** – double *Input*

*On entry:* the argument for which **ucv** must be evaluated.

2: **comm** – Nag\_Comm \*

Pointer to structure of type Nag\_Comm; the following members are relevant to **ucv**.

**user** – double \*  
**iuser** – Integer \*  
**p** – Pointer

The type Pointer will be `void *`. Before calling `nag_robust_m_regsn_wts` (g02hbc) you may allocate memory and initialize these pointers with various quantities for use by **ucv** when called from `nag_robust_m_regsn_wts` (g02hbc) (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of observations.  
*Constraint:*  $n > 1$ .
- 4: **m** – Integer *Input*  
*On entry:*  $m$ , the number of independent variables.  
*Constraint:*  $1 \leq m \leq n$ .
- 5: **x[dim]** – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **x** must be at least  
 $\max(1, \mathbf{pdx} \times \mathbf{m})$  when **order** = Nag\_ColMajor;  
 $\max(1, n \times \mathbf{pdx})$  when **order** = Nag\_RowMajor.  
Where  $\mathbf{X}(i, j)$  appears in this document, it refers to the array element  
 $\mathbf{x}[(j-1) \times \mathbf{pdx} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{x}[(i-1) \times \mathbf{pdx} + j - 1]$  when **order** = Nag\_RowMajor.  
*On entry:* the real matrix  $X$ , i.e., the independent variables.  $\mathbf{X}(i, j)$  must contain the  $ij$ th element of **x**, for  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, m$ .
- 6: **pdx** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **x**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  $\mathbf{pdx} \geq n$ ;  
if **order** = Nag\_RowMajor,  $\mathbf{pdx} \geq m$ .
- 7: **a[m × (m + 1)/2]** – double *Input/Output*  
*On entry:* an initial estimate of the lower triangular real matrix  $A$ . Only the lower triangular elements must be given and these should be stored row-wise in the array.  
The diagonal elements must be  $\neq 0$ , although in practice will usually be  $> 0$ . If the magnitudes of the columns of  $X$  are of the same order the identity matrix will often provide a suitable initial value for  $A$ . If the columns of  $X$  are of different magnitudes, the diagonal elements of the initial value of  $A$  should be approximately inversely proportional to the magnitude of the columns of  $X$ .  
*On exit:* the lower triangular elements of the matrix  $A$ , stored row-wise.
- 8: **z[n]** – double *Output*  
*On exit:* the value  $\|z_i\|_2$ , for  $i = 1, 2, \dots, n$ .
- 9: **bl** – double *Input*  
*On entry:* the magnitude of the bound for the off-diagonal elements of  $S_k$ .

*Suggested value:* **bl** = 0.9.

*Constraint:* **bl** > 0.0.

10: **bd** – double *Input*

*On entry:* the magnitude of the bound for the diagonal elements of  $S_k$ .

*Suggested value:* **bd** = 0.9.

*Constraint:* **bd** > 0.0.

11: **tol** – double *Input*

*On entry:* the relative precision for the final value of  $A$ . Iteration will stop when the maximum value of  $|s_{jl}|$  is less than **tol**.

*Constraint:* **tol** > 0.0.

12: **maxit** – Integer *Input*

*On entry:* the maximum number of iterations that will be used during the calculation of  $A$ .

A value of **maxit** = 50 will often be adequate.

*Constraint:* **maxit** > 0.

13: **nitmon** – Integer *Input*

*On entry:* determines the amount of information that is printed on each iteration.

**nitmon** > 0

The value of  $A$  and the maximum value of  $|s_{jl}|$  will be printed at the first and every **nitmon** iterations.

**nitmon** ≤ 0

No iteration monitoring is printed.

14: **outfile** – const char \* *Input*

*On entry:* a null terminated character string giving the name of the file to which results should be printed. If **outfile** = **NULL** or an empty string then the `stdout` stream is used. Note that the file will be opened in the append mode.

15: **nit** – Integer \* *Output*

*On exit:* the number of iterations performed.

16: **comm** – Nag\_Comm \*

The NAG communication argument (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

17: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_CONVERGENCE**

Iterations to calculate weights failed to converge in **maxit** iterations: **maxit** =  $\langle value \rangle$ .

**NE\_FUN\_RET\_VAL**

Value returned by **ucv** function  $< 0$ :  $u(\langle value \rangle) = \langle value \rangle$ .

**NE\_INT**

On entry, **maxit** =  $\langle value \rangle$ .

Constraint: **maxit**  $> 0$ .

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $> 1$ .

On entry, **pdx** =  $\langle value \rangle$ .

Constraint: **pdx**  $> 0$ .

**NE\_INT\_2**

On entry, **m** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint:  $1 \leq \mathbf{m} \leq \mathbf{n}$ .

On entry, **pdx** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .

Constraint: **pdx**  $\geq \mathbf{m}$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_CLOSE\_FILE**

Cannot close file  $\langle value \rangle$ .

**NE\_NOT\_WRITE\_FILE**

Cannot open file  $\langle value \rangle$  for writing.

**NE\_REAL**

On entry, **bd** =  $\langle value \rangle$ .

Constraint: **bd**  $> 0.0$ .

On entry, **bl** =  $\langle value \rangle$ .

Constraint: **bl**  $> 0.0$ .

On entry, **tol** =  $\langle value \rangle$ .

Constraint: **tol**  $> 0.0$ .

**NE\_ZERO\_DIAGONAL**

On entry, diagonal element  $\langle value \rangle$  of **a** is 0.

## 7 Accuracy

On successful exit the accuracy of the results is related to the value of **tol**; see Section 5.

## 8 Parallelism and Performance

nag\_robust\_m\_regsn\_wts (g02hbc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The existence of  $A$  will depend upon the function  $u$ ; (see Hampel *et al.* (1986) and Marazzi (1987)), also if  $X$  is not of full rank a value of  $A$  will not be found. If the columns of  $X$  are almost linearly related then convergence will be slow.

## 10 Example

This example reads in a matrix of real numbers and computes the Krasker–Welsch weights (see Marazzi (1987)). The matrix  $A$  and the weights are then printed.

### 10.1 Program Text

```
/* nag_robust_m_regsn_wts (g02hbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <math.h>
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>
#include <nags.h>
#include <nagx01.h>
#include <nagx02.h>

#ifdef __cplusplus
extern "C"
{
#endif
    static double NAG_CALL ucv(double t, Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

int main(void)
{
    /* Scalars */
    double bd, bl, tol;
    Integer exit_status, i, j, k, l1, l2, m, maxit, mm, n, nit, nitmon;
    Integer pdx;
    NagError fail;
    Nag_OrderType order;
    Nag_Comm comm;
```

```

/* Arrays */
static double ruser[1] = { -1.0 };
double *a = 0, *x = 0, *z = 0;

#ifdef NAG_COLUMN_MAJOR
#define X(I, J) x[(J-1)*pdx + I - 1]
    order = Nag_ColMajor;
#else
#define X(I, J) x[(I-1)*pdx + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    exit_status = 0;
    printf("nag_robust_m_regsn_wts (g02hbc) Example Program Results\n");

/* For communication with user-supplied functions: */
comm.user = ruser;

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Read in the dimensions of X */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &m);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &m);
#endif
/* Allocate memory */
if (!(a = NAG_ALLOC(m * (m + 1) / 2, double)) ||
    !(x = NAG_ALLOC(n * m, double)) || !(z = NAG_ALLOC(n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

#ifdef NAG_COLUMN_MAJOR
    pdx = n;
#else
    pdx = m;
#endif

/* Read in the X matrix */
for (i = 1; i <= n; ++i) {
    for (j = 1; j <= m; ++j)
#ifdef _WIN32
        scanf_s("%lf", &X(i, j));
#else
        scanf("%lf", &X(i, j));
#endif
}
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Read in the initial value of A */
mm = (m + 1) * m / 2;
for (j = 1; j <= mm; ++j)
#ifdef _WIN32
    scanf_s("%lf", &a[j - 1]);
#else
    scanf("%lf", &a[j - 1]);
#endif
#ifdef _WIN32

```

```

    scanf_s("%*[^\\n] ");
#else
    scanf("%*[^\\n] ");
#endif

    /* Set the values remaining parameters */
    bl = 0.9;
    bd = 0.9;
    maxit = 50;
    tol = 5e-5;
    /* Change nitmon to a positive value if monitoring information
     * is required
     */
    nitmon = 0;
    /* nag_robust_m_regsn_wts (g02hbc).
     * Robust regression, compute weights for use with
     * nag_robust_m_regsn_user_fn (g02hdc)
     */
    nag_robust_m_regsn_wts(order, ucv, n, m, x, pdx, a, z, bl, bd, tol, maxit,
                          nitmon, 0, &nit, &comm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_robust_m_regsn_wts (g02hbc).\\n%s\\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("nag_robust_m_regsn_wts (g02hbc) required %4" NAG_IFMT
          " iterations to " "converge\\n\\n", nit);
    printf("Matrix A\\n");
    l2 = 0;
    for (j = 1; j <= m; ++j) {
        l1 = l2 + 1;
        l2 += j;
        for (k = l1; k <= l2; ++k)
            printf("%9.4f%s", a[k - 1], k % 6 == 0 || k == l2 ? "\\n" : " ");
    }

    printf("\\n");
    printf("Vector Z\\n");
    for (i = 1; i <= n; ++i)
        printf("%9.4f\\n", z[i - 1]);

    /* Calculate Krasker-Welsch weights */
    printf("\\n");
    printf("Vector of weights\\n");
    for (i = 1; i <= n; ++i) {
        z[i - 1] = 1.0 / z[i - 1];
        printf("%9.4f\\n", z[i - 1]);
    }

END:
    NAG_FREE(a);
    NAG_FREE(x);
    NAG_FREE(z);

    return exit_status;
}

static double NAG_CALL ucv(double t, Nag_Comm *comm)
{
    /* Scalars */
    double pc, pd, q, q2;
    double ret_val;

    /* ucv function for Krasker-Welsch weights */
    if (comm->user[0] == -1.0) {
        printf("(User-supplied callback ucv, first invocation.)\\n");
        comm->user[0] = 0.0;
    }
    ret_val = 1.0;
    if (t != 0.0) {

```



```

    q = 2.5 / t;
    q2 = q * q;
    /* nag_cumul_normal (s15abc).
     * Cumulative Normal distribution function P(x)
     */
    pc = nag_cumul_normal(q);
    /* nag_real_smallest_number (x02akc).
     * The smallest positive model number
     */
    if (q2 < -log(nag_real_smallest_number))
        /* nag_pi (x01aac).
         * pi
         */
        pd = exp(-q2 / 2.0) / sqrt(nag_pi * 2.0);
    else
        pd = 0.0;
    ret_val = (pc * 2.0 - 1.0) * (1.0 - q2) + q2 - q * 2.0 * pd;
}
return ret_val;
}

```

## 10.2 Program Data

nag\_robust\_m\_regsn\_wts (g02hbc) Example Program Data

```

      5      3              : N  M

1.0 -1.0 -1.0              : X1  X2  X3
1.0 -1.0  1.0
1.0  1.0 -1.0
1.0  1.0  1.0
1.0  0.0  3.0              : End of X1 X2 and X3 values

1.0 0.0 1.0 0.0 0.0 1.0   : A

```

## 10.3 Program Results

nag\_robust\_m\_regsn\_wts (g02hbc) Example Program Results

(User-supplied callback ucv, first invocation.)

nag\_robust\_m\_regsn\_wts (g02hbc) required 16 iterations to converge

Matrix A

```

  1.3208
  0.0000  1.4518
-0.5753  0.0000  0.9340

```

Vector Z

```

  2.4760
  1.9953
  2.4760
  1.9953
  2.5890

```

Vector of weights

```

  0.4039
  0.5012
  0.4039
  0.5012
  0.3862

```

---