

## NAG Library Function Document

### nag\_real\_symm\_sparse\_eigensystem\_sol (f12fcc)

**Note:** this function uses **optional parameters** to define choices in the problem specification. If you wish to use default settings for all of the optional parameters, then the option setting function `nag_real_symm_sparse_eigensystem_option (f12fdc)` need not be called. If, however, you wish to reset some or all of the settings please refer to Section 11 in `nag_real_symm_sparse_eigensystem_option (f12fdc)` for a detailed description of the specification of the optional parameters.

## 1 Purpose

`nag_real_symm_sparse_eigensystem_sol (f12fcc)` is a post-processing function in a suite of functions which includes `nag_real_symm_sparse_eigensystem_init (f12fac)`, `nag_real_symm_sparse_eigensystem_iter (f12fbc)`, `nag_real_symm_sparse_eigensystem_option (f12fdc)` and `nag_real_symm_sparse_eigensystem_monit (f12fec)`. `nag_real_symm_sparse_eigensystem_sol (f12fcc)` must be called following a final exit from `nag_real_symm_sparse_eigensystem_iter (f12fbc)`.

## 2 Specification

```
#include <nag.h>
#include <nagf12.h>

void nag_real_symm_sparse_eigensystem_sol (Integer *nconv, double d[],
double z[], double sigma, const double resid[], double v[],
double comm[], Integer icomm[], NagError *fail)
```

## 3 Description

The suite of functions is designed to calculate some of the eigenvalues,  $\lambda$ , (and optionally the corresponding eigenvectors,  $x$ ) of a standard eigenvalue problem  $Ax = \lambda x$ , or of a generalized eigenvalue problem  $Ax = \lambda Bx$  of order  $n$ , where  $n$  is large and the coefficient matrices  $A$  and  $B$  are sparse, real and symmetric. The suite can also be used to find selected eigenvalues/eigenvectors of smaller scale dense, real and symmetric problems.

Following a call to `nag_real_symm_sparse_eigensystem_iter (f12fbc)`, `nag_real_symm_sparse_eigensystem_sol (f12fcc)` returns the converged approximations to eigenvalues and (optionally) the corresponding approximate eigenvectors and/or an orthonormal basis for the associated approximate invariant subspace. The eigenvalues (and eigenvectors) are selected from those of a standard or generalized eigenvalue problem defined by real symmetric matrices. There is negligible additional cost to obtain eigenvectors; an orthonormal basis is always computed, but there is an additional storage cost if both are requested.

`nag_real_symm_sparse_eigensystem_sol (f12fcc)` is based on the function **dseupd** from the ARPACK package, which uses the Implicitly Restarted Lanczos iteration method. The method is described in Lehoucq and Sorensen (1996) and Lehoucq (2001) while its use within the ARPACK software is described in great detail in Lehoucq *et al.* (1998). An evaluation of software for computing eigenvalues of sparse symmetric matrices is provided in Lehoucq and Scott (1996). This suite of functions offers the same functionality as the ARPACK software for real symmetric problems, but the interface design is quite different in order to make the option setting clearer and to simplify some of the interfaces.

`nag_real_symm_sparse_eigensystem_sol (f12fcc)`, is a post-processing function that must be called following a successful final exit from `nag_real_symm_sparse_eigensystem_iter (f12fbc)`. `nag_real_symm_sparse_eigensystem_sol (f12fcc)` uses data returned from `nag_real_symm_sparse_eigensystem_iter (f12fbc)` and options, set either by default or explicitly by calling `nag_real_symm_sparse_eigensystem_option (f12fdc)`, to return the converged approximations to selected eigenvalues and (optionally):

- the corresponding approximate eigenvectors;
- an orthonormal basis for the associated approximate invariant subspace;
- both.

## 4 References

Lehoucq R B (2001) Implicitly restarted Arnoldi methods and subspace iteration *SIAM Journal on Matrix Analysis and Applications* **23** 551–562

Lehoucq R B and Scott J A (1996) An evaluation of software for computing eigenvalues of sparse nonsymmetric matrices *Preprint MCS-P547-1195* Argonne National Laboratory

Lehoucq R B and Sorensen D C (1996) Deflation techniques for an implicitly restarted Arnoldi iteration *SIAM Journal on Matrix Analysis and Applications* **17** 789–821

Lehoucq R B, Sorensen D C and Yang C (1998) *ARPACK Users' Guide: Solution of Large-scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods* SIAM, Philadelphia

## 5 Arguments

- 1: **nconv** – Integer \* *Output*  
*On exit:* the number of converged eigenvalues as found by nag\_real\_symm\_sparse\_eigensystem\_iter (f12fbc).
- 2: **d**[*dim*] – double *Output*  
**Note:** the dimension, *dim*, of the array **d** must be at least **ncv** (see nag\_real\_symm\_sparse\_eigensystem\_init (f12fac)).  
*On exit:* the first **nconv** locations of the array **d** contain the converged approximate eigenvalues.
- 3: **z**[**n** × (**ncv** + 1)] – double *Output*  
*On exit:* if the default option **Vectors** = RITZ (see nag\_real\_symm\_sparse\_eigensystem\_option (f12fdc)) has been selected then **z** contains the final set of eigenvectors corresponding to the eigenvalues held in **d**. The real eigenvector associated with an eigenvalue is stored in the corresponding array section of **z**.
- 4: **sigma** – double *Input*  
*On entry:* if one of the **Shifted Inverse** (see nag\_real\_symm\_sparse\_eigensystem\_option (f12fdc)) modes has been selected then **sigma** contains the real shift used; otherwise **sigma** is not referenced.
- 5: **resid**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **resid** must be at least **n** (see nag\_real\_symm\_sparse\_eigensystem\_init (f12fac)).  
*On entry:* must not be modified following a call to nag\_real\_symm\_sparse\_eigensystem\_iter (f12fbc) since it contains data required by nag\_real\_symm\_sparse\_eigensystem\_sol (f12fcc).
- 6: **v**[**n** × **ncv**] – double *Input/Output*  
The *i*th element of the *j*th basis vector is stored in location **v**[**n** × (*j* – 1) + *i* – 1], for *i* = 1, 2, ..., **n** and *j* = 1, 2, ..., **ncv**.  
*On entry:* the **ncv** sections of **v**, of length *n*, contain the Lanczos basis vectors for OP as constructed by nag\_real\_symm\_sparse\_eigensystem\_iter (f12fbc).

*On exit:* if the option **Vectors** = SCHUR has been set, or the option **Vectors** = RITZ has been set and a separate array **z** has been passed (i.e., **z** does not equal **v**), then the first **nconv** sections of **v**, of length *n*, will contain approximate Schur vectors that span the desired invariant subspace.

7: **comm**[*dim*] – double *Communication Array*

**Note:** the dimension, *dim*, of the array **comm** must be at least  $\max(1, \mathbf{lcomm})$  (see `nag_real_symm_sparse_eigensystem_init` (f12fac)).

*On initial entry:* must remain unchanged from the prior call to `nag_real_symm_sparse_eigensystem_init` (f12fac).

*On exit:* contains data on the current state of the solution.

8: **icomm**[*dim*] – Integer *Communication Array*

**Note:** the dimension, *dim*, of the array **icomm** must be at least  $\max(1, \mathbf{licomm})$  (see `nag_real_symm_sparse_eigensystem_init` (f12fac)).

*On initial entry:* must remain unchanged from the prior call to `nag_real_symm_sparse_eigensystem_init` (f12fac).

*On exit:* contains data on the current state of the solution.

9: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument *⟨value⟩* had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_INVALID\_OPTION

On entry, **Vectors** = SELECT, but this is not yet implemented.

### NE\_MAX\_ITER

During calculation of a tridiagonal form, there was a failure to compute *⟨value⟩* eigenvalues in a total of *⟨value⟩* iterations.

### NE\_MISSING\_CALL

Either the function was called out of sequence (following an initial call to the setup function and following completion of calls to the reverse communication function) or the communication arrays have become corrupted.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_RITZ\_COUNT**

Got a different count of the number of converged Ritz values than the value passed to it through the argument **icomm**: number counted =  $\langle value \rangle$ , number expected =  $\langle value \rangle$ . This usually indicates that a communication array has been altered or has become corrupted between calls to `nag_real_symm_sparse_eigensystem_iter` (f12fbc) and `nag_real_symm_sparse_eigensystem_sol` (f12fcc).

**NE\_ZERO\_EIGS\_FOUND**

The number of eigenvalues found to sufficient accuracy, as communicated through the argument **icomm**, is zero. You should experiment with different values of **nev** and **ncv**, or select a different computational mode or increase the maximum number of iterations prior to calling `nag_real_symm_sparse_eigensystem_iter` (f12fbc).

**7 Accuracy**

The relative accuracy of a Ritz value,  $\lambda$ , is considered acceptable if its Ritz estimate  $\leq \mathbf{Tolerance} \times |\lambda|$ . The default **Tolerance** used is the *machine precision* given by `nag_machine_precision` (X02AJC).

**8 Parallelism and Performance**

`nag_real_symm_sparse_eigensystem_sol` (f12fcc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_real_symm_sparse_eigensystem_sol` (f12fcc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

None.

**10 Example**

This example solves  $Ax = \lambda Bx$  in regular mode, where  $A$  and  $B$  are obtained from the standard central difference discretization of the one-dimensional Laplacian operator  $\frac{d^2 u}{dx^2}$  on  $[0, 1]$ , with zero Dirichlet boundary conditions.

**10.1 Program Text**

```
/* nag_real_symm_sparse_eigensystem_sol (f12fcc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
```

```

#include <stdio.h>
#include <nagf12.h>
#include <nagf16.h>

static void av(Integer, double *);
static void mv(Integer, double *, double *);
static void my_dgttrf(Integer, double *, double *, double *, double *,
                     double *, Integer *, Integer *);
static void my_dgttrs(Integer, double *, double *, double *,
                     double *, Integer *, double *, double *);

int main(void)
{
    /* Constants */
    Integer licomm = 140, imon = 0;

    /* Scalars */
    double estnrm, h, r1, r2, sigma = 0.0;
    Integer exit_status, info, irevcn, j, lcomm, n, nconv, ncv;
    Integer nev, niter, nshift;
    /* Nag types */
    NagError fail;
    /* Arrays */
    double *dd = 0, *dl = 0, *du = 0, *du2 = 0, *comm = 0, *eigest = 0;
    double *eigv = 0, *resid = 0, *v = 0;
    Integer *icomm = 0, *ipiv = 0;
    /* Pointers */
    double *mx = 0, *x = 0, *y = 0;

    exit_status = 0;
    INIT_FAIL(fail);

    printf("nag_real_symm_sparse_eigensystem_sol (f12fcc) Example "
           "Program Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Read values for nx, nev and cnv from data file. */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &nev, &ncv);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &nev, &ncv);
#endif

    /* Allocate memory */
    lcomm = 3 * n + ncv * ncv + 8 * ncv + 60;
    if (!(dd = NAG_ALLOC(n, double)) ||
        !(dl = NAG_ALLOC(n, double)) ||
        !(du = NAG_ALLOC(n, double)) ||
        !(du2 = NAG_ALLOC(n, double)) ||
        !(comm = NAG_ALLOC(lcomm, double)) ||
        !(eigv = NAG_ALLOC(ncv, double)) ||
        !(eigest = NAG_ALLOC(ncv, double)) ||
        !(resid = NAG_ALLOC(n, double)) ||
        !(v = NAG_ALLOC(n * ncv, double)) ||
        !(icomm = NAG_ALLOC(licomm, Integer)) ||
        !(ipiv = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    /* Initialize communication arrays for problem using
       nag_real_symm_sparse_eigensystem_init (f12fac). */
    nag_real_symm_sparse_eigensystem_init(n, nev, ncv, icomm, licomm, comm,
                                          lcomm, &fail);
    if (fail.code != NE_NOERROR) {

```

```

    printf("Error from nag_real_symm_sparse_eigensystem_init "
           "(f12fac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Select the problem type using
   nag_real_symm_sparse_eigensystem_option (f12fdc). */
nag_real_symm_sparse_eigensystem_option("generalized", icomm, comm, &fail);
if (fail.code != NE_NOERROR) {
    printf(" Error from nag_real_symm_sparse_eigensystem_option "
           "(f12fdc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Setup M and factorise */
h = 1.0 / (double) (n + 1);
r1 = 2.0 * h / 3.0;
r2 = h / 6.0;
for (j = 0; j <= n - 1; ++j) {
    dd[j] = r1;
    dl[j] = r2;
    du[j] = r2;
}
my_dgtrrf(n, dl, dd, du, du2, ipiv, &info);

irevcm = 0;
REVCOMLOOP:
/* Repeated calls to reverse communication routine
   nag_real_symm_sparse_eigensystem_iter (f12fbc). */
nag_real_symm_sparse_eigensystem_iter(&irevcm, resid, v, &x, &y, &mx,
                                     &nshift, comm, icomm, &fail);
if (irevcm != 5) {
    if (irevcm == -1 || irevcm == 1) {
        /* Perform the following operations in order:
           x <--- A*x;
           y <--- inv[M]*x. */
        av(n, x);
        my_dgttrs(n, dl, dd, du, du2, ipiv, x, y);
    }
    else if (irevcm == 2) {
        /* Perform y <--- M*x. */
        mv(n, x, y);
    }
    else if (irevcm == 4 && imon == 1) {
        /* If imon=1, get monitoring information using
           nag_real_symm_sparse_eigensystem_monit (f12fec). */
        nag_real_symm_sparse_eigensystem_monit(&niter, &nconv, eigv, eigst,
                                              icomm, comm);

        /* Compute 2-norm of Ritz estimates using
           nag_dge_norm (f16rac). */
        nag_dge_norm(Nag_ColMajor, Nag_FrobeniusNorm, nev, 1, eigst, nev,
                    &estnrm, &fail);
        printf("Iteration %3" NAG_IFMT " ", niter);
        printf(" No. converged = %3" NAG_IFMT " ", nconv);
        printf(" norm of estimates = %17.8e\n", estnrm);
    }
    goto REVCOMLOOP;
}
if (fail.code == NE_NOERROR) {
    /* Post-Process using nag_real_symm_sparse_eigensystem_sol (f12fcc)
       to compute eigenvalues/vectors. */
    nag_real_symm_sparse_eigensystem_sol(&nconv, eigv, v, sigma, resid, v,
                                       comm, icomm, &fail);
    printf("\n The %4" NAG_IFMT " generalized Ritz values", nconv);
    printf(" of largest magnitude are:\n\n");
    for (j = 0; j <= nconv - 1; ++j) {
        printf("%8" NAG_IFMT "%5s%9.1f\n", j + 1, "", eigv[j]);
    }
}
else {

```

```

    printf(" Error from nag_real_symm_sparse_eigensystem_iter "
           "(f12fbc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
    NAG_FREE(dd);
    NAG_FREE(dl);
    NAG_FREE(du);
    NAG_FREE(du2);
    NAG_FREE(comm);
    NAG_FREE(eigv);
    NAG_FREE(eigest);
    NAG_FREE(resid);
    NAG_FREE(v);
    NAG_FREE(icom);
    NAG_FREE(ipiv);

    return exit_status;
}

static void mv(Integer n, double *v, double *y)
{
    /* Scalars */
    double h;
    Integer j;

    /* Function Body */
    h = 1.0 / ((double) (n + 1) * 6.0);
    y[0] = h * (v[0] * 4.0 + v[1]);
    for (j = 1; j <= n - 2; ++j) {
        y[j] = h * (v[j - 1] + v[j] * 4.0 + v[j + 1]);
    }
    y[n - 1] = h * (v[n - 2] + v[n - 1] * 4.0);
} /* mv */

static void av(Integer n, double *v)
{
    /* Scalars */
    double h, vj, vjml;
    Integer j;

    /* Function Body */
    h = (double) (n + 1);
    vjml = v[0];
    v[0] = h * (vjml * 2.0 - v[1]);
    for (j = 1; j <= n - 2; ++j) {
        vj = v[j];
        v[j] = h * (-vjml + vj * 2.0 - v[j + 1]);
        vjml = vj;
    }
    v[n - 1] = h * (-vjml + v[n - 1] * 2.0);
} /* av */

static void my_dgttrf(Integer n, double dl[], double d[],
                     double du[], double du2[], Integer ipiv[],
                     Integer *info)
{
    /* A simple C version of the Lapack routine dgttrf with argument
       checking removed */
    /* Scalars */
    double temp, fact;
    Integer i;
    /* Function Body */
    *info = 0;
    for (i = 0; i < n; ++i) {
        ipiv[i] = i;
    }
    for (i = 0; i < n - 2; ++i) {
        du2[i] = 0.0;
    }
}

```

```

for (i = 0; i < n - 2; i++) {
    if (fabs(d[i]) >= fabs(dl[i])) {
        /* No row interchange required, eliminate dl[i]. */
        if (d[i] != 0.0) {
            fact = dl[i] / d[i];
            dl[i] = fact;
            d[i + 1] = d[i + 1] - fact * du[i];
        }
    }
    else {
        /* Interchange rows I and I+1, eliminate dl[I] */
        fact = d[i] / dl[i];
        d[i] = dl[i];
        dl[i] = fact;
        temp = du[i];
        du[i] = d[i + 1];
        d[i + 1] = temp - fact * d[i + 1];
        du2[i] = du[i + 1];
        du[i + 1] = -fact * du[i + 1];
        ipiv[i] = i + 1;
    }
}
if (n > 1) {
    i = n - 2;
    if (fabs(d[i]) >= fabs(dl[i])) {
        if (d[i] != 0.0) {
            fact = dl[i] / d[i];
            dl[i] = fact;
            d[i + 1] = d[i + 1] - fact * du[i];
        }
    }
    else {
        fact = d[i] / dl[i];
        d[i] = dl[i];
        dl[i] = fact;
        temp = du[i];
        du[i] = d[i + 1];
        d[i + 1] = temp - fact * d[i + 1];
        ipiv[i] = i + 1;
    }
}
/* Check for a zero on the diagonal of U. */
for (i = 0; i < n; ++i) {
    if (d[i] == 0.0) {
        *info = i;
        goto END;
    }
}
END:
return;
}

static void my_dgttrs(Integer n, double dl[], double d[],
                     double du[], double du2[], Integer ipiv[],
                     double b[], double y[])
{
    /* A simple C version of the Lapack routine dgttrs with argument
       checking removed, the number of right-hand-sides=1, Trans='N' */
    /* Scalars */
    Integer i, ip;
    double temp;
    /* Solve L*x = b. */
    for (i = 0; i <= n - 1; ++i) {
        y[i] = b[i];
    }
    for (i = 0; i < n - 1; ++i) {
        ip = ipiv[i];
        temp = y[i + 1 - ip + i] - dl[i] * y[ip];
        y[i] = y[ip];
        y[i + 1] = temp;
    }
}

```



```

/* Solve U*x = b. */
y[n - 1] = y[n - 1] / d[n - 1];
if (n > 1) {
    y[n - 2] = (y[n - 2] - du[n - 2] * y[n - 1]) / d[n - 2];
}
for (i = n - 3; i >= 0; --i) {
    y[i] = (y[i] - du[i] * y[i + 1] - du2[i] * y[i + 2]) / d[i];
}
return;
}

```

## 10.2 Program Data

```

nag_real_symm_sparse_eigensystem_sol (f12fcc) Example Program Data
100  4  10 : Values for n, nev and ncv

```

## 10.3 Program Results

```

nag_real_symm_sparse_eigensystem_sol (f12fcc) Example Program Results

```

The 4 generalized Ritz values of largest magnitude are:

1	121003.5
2	121616.6
3	122057.5
4	122323.2

---