

# NAG Library Function Document

## nag\_dormhr (f08ngc)

### 1 Purpose

nag\_dormhr (f08ngc) multiplies an arbitrary real matrix  $C$  by the real orthogonal matrix  $Q$  which was determined by nag\_dgehrd (f08nec) when reducing a real general matrix to Hessenberg form.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_dormhr (Nag_OrderType order, Nag_SideType side,
                 Nag_TransType trans, Integer m, Integer n, Integer ilo, Integer ihi,
                 const double a[], Integer pda, const double tau[], double c[],
                 Integer pdc, NagError *fail)
```

### 3 Description

nag\_dormhr (f08ngc) is intended to be used following a call to nag\_dgehrd (f08nec), which reduces a real general matrix  $A$  to upper Hessenberg form  $H$  by an orthogonal similarity transformation:  $A = QHQ^T$ . nag\_dgehrd (f08nec) represents the matrix  $Q$  as a product of  $i_{hi} - i_{lo}$  elementary reflectors. Here  $i_{lo}$  and  $i_{hi}$  are values determined by nag\_dgebal (f08nhc) when balancing the matrix; if the matrix has not been balanced,  $i_{lo} = 1$  and  $i_{hi} = n$ .

This function may be used to form one of the matrix products

$$QC, Q^TC, CQ \text{ or } CQ^T,$$

overwriting the result on  $C$  (which may be any real rectangular matrix).

A common application of this function is to transform a matrix  $V$  of eigenvectors of  $H$  to the matrix  $QV$  of eigenvectors of  $A$ .

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **side** – Nag\_SideType *Input*

*On entry:* indicates how  $Q$  or  $Q^T$  is to be applied to  $C$ .

**side** = Nag\_LeftSide

$Q$  or  $Q^T$  is applied to  $C$  from the left.

**side** = Nag\_RightSide

$Q$  or  $Q^T$  is applied to  $C$  from the right.

*Constraint:* **side** = Nag\_LeftSide or Nag\_RightSide.

3: **trans** – Nag\_TransType

*Input*

*On entry:* indicates whether  $Q$  or  $Q^T$  is to be applied to  $C$ .

**trans** = Nag\_NoTrans

$Q$  is applied to  $C$ .

**trans** = Nag\_Trans

$Q^T$  is applied to  $C$ .

*Constraint:* **trans** = Nag\_NoTrans or Nag\_Trans.

4: **m** – Integer

*Input*

*On entry:*  $m$ , the number of rows of the matrix  $C$ ;  $m$  is also the order of  $Q$  if **side** = Nag\_LeftSide.

*Constraint:* **m**  $\geq 0$ .

5: **n** – Integer

*Input*

*On entry:*  $n$ , the number of columns of the matrix  $C$ ;  $n$  is also the order of  $Q$  if **side** = Nag\_RightSide.

*Constraint:* **n**  $\geq 0$ .

6: **ilo** – Integer

*Input*

7: **ihi** – Integer

*Input*

*On entry:* these **must** be the same arguments **ilo** and **ihi**, respectively, as supplied to nag\_dgehrd (f08nec).

*Constraints:*

if **side** = Nag\_LeftSide and **m**  $> 0$ ,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{m}$ ;  
 if **side** = Nag\_LeftSide and **m** = 0, **ilo** = 1 and **ihi** = 0;  
 if **side** = Nag\_RightSide and **n**  $> 0$ ,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{n}$ ;  
 if **side** = Nag\_RightSide and **n** = 0, **ilo** = 1 and **ihi** = 0.

8: **a**[*dim*] – const double

*Input*

**Note:** the dimension, *dim*, of the array **a** must be at least

$\max(1, \mathbf{pda} \times \mathbf{m})$  when **side** = Nag\_LeftSide;

$\max(1, \mathbf{pda} \times \mathbf{n})$  when **side** = Nag\_RightSide.

*On entry:* details of the vectors which define the elementary reflectors, as returned by nag\_dgehrd (f08nec).

9: **pda** – Integer

*Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.

*Constraints:*

if **side** = Nag\_LeftSide, **pda**  $\geq \max(1, \mathbf{m})$ ;  
 if **side** = Nag\_RightSide, **pda**  $\geq \max(1, \mathbf{n})$ .

- 10: **tau**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **tau** must be at least  
 $\max(1, \mathbf{m} - 1)$  when **side** = Nag\_LeftSide;  
 $\max(1, \mathbf{n} - 1)$  when **side** = Nag\_RightSide.  
*On entry:* further details of the elementary reflectors, as returned by nag\_dgehrd (f08nec).
- 11: **c**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **c** must be at least  
 $\max(1, \mathbf{pdc} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pdc})$  when **order** = Nag\_RowMajor.  
The (*i*, *j*)th element of the matrix *C* is stored in  
 $\mathbf{c}[(j - 1) \times \mathbf{pdc} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{c}[(i - 1) \times \mathbf{pdc} + j - 1]$  when **order** = Nag\_RowMajor.  
*On entry:* the *m* by *n* matrix *C*.  
*On exit:* **c** is overwritten by  $QC$  or  $Q^T C$  or  $CQ$  or  $CQ^T$  as specified by **side** and **trans**.
- 12: **pdc** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **c**.  
*Constraints:*  
if **order** = Nag\_ColMajor, **pdc**  $\geq \max(1, \mathbf{m})$ ;  
if **order** = Nag\_RowMajor, **pdc**  $\geq \max(1, \mathbf{n})$ .
- 13: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).
- ## 6 Error Indicators and Warnings
- ### NE\_ALLOC\_FAIL
- Dynamic memory allocation failed.  
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.
- ### NE\_BAD\_PARAM
- On entry, argument *⟨value⟩* had an illegal value.
- ### NE\_ENUM\_INT\_3
- On entry, **side** = *⟨value⟩*, **m** = *⟨value⟩*, **n** = *⟨value⟩* and **pda** = *⟨value⟩*.  
Constraint: if **side** = Nag\_LeftSide, **pda**  $\geq \max(1, \mathbf{m})$ ;  
if **side** = Nag\_RightSide, **pda**  $\geq \max(1, \mathbf{n})$ .  
On entry, **side** = *⟨value⟩*, **pda** = *⟨value⟩*, **m** = *⟨value⟩* and **n** = *⟨value⟩*.  
Constraint: if **side** = Nag\_LeftSide, **pda**  $\geq \max(1, \mathbf{m})$ ;  
if **side** = Nag\_RightSide, **pda**  $\geq \max(1, \mathbf{n})$ .
- ### NE\_ENUM\_INT\_4
- On entry, **side** = *⟨value⟩*, **m** = *⟨value⟩*, **n** = *⟨value⟩*, **ilo** = *⟨value⟩* and **ihi** = *⟨value⟩*.  
Constraint: if **side** = Nag\_LeftSide and **m** > 0,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{m}$ ;  
if **side** = Nag\_LeftSide and **m** = 0, **ilo** = 1 and **ihi** = 0;

if **side** = Nag\_RightSide and **n** > 0,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{n}$ ;  
 if **side** = Nag\_RightSide and **n** = 0, **ilo** = 1 and **ihi** = 0.

#### NE\_INT

On entry, **m** =  $\langle value \rangle$ .  
 Constraint: **m**  $\geq$  0.

On entry, **n** =  $\langle value \rangle$ .  
 Constraint: **n**  $\geq$  0.

On entry, **pda** =  $\langle value \rangle$ .  
 Constraint: **pda** > 0.

On entry, **pdc** =  $\langle value \rangle$ .  
 Constraint: **pdc** > 0.

#### NE\_INT\_2

On entry, **pdc** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .  
 Constraint: **pdc**  $\geq$  max(1, **m**).

On entry, **pdc** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pdc**  $\geq$  max(1, **n**).

#### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

#### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed result differs from the exact result by a matrix  $E$  such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where  $\epsilon$  is the *machine precision*.

## 8 Parallelism and Performance

nag\_dormhr (f08ngc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dormhr (f08ngc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately  $2nq^2$  if **side** = Nag\_LeftSide and  $2mq^2$  if **side** = Nag\_RightSide, where  $q = i_{hi} - i_{lo}$ .

The complex analogue of this function is nag\_zunmhr (f08nuc).

## 10 Example

This example computes all the eigenvalues of the matrix  $A$ , where

$$A = \begin{pmatrix} 0.35 & 0.45 & -0.14 & -0.17 \\ 0.09 & 0.07 & -0.54 & 0.35 \\ -0.44 & -0.33 & -0.03 & 0.17 \\ 0.25 & -0.32 & -0.13 & 0.11 \end{pmatrix},$$

and those eigenvectors which correspond to eigenvalues  $\lambda$  such that  $\text{Re}(\lambda) < 0$ . Here  $A$  is general and must first be reduced to upper Hessenberg form  $H$  by `nag_dgehrd` (f08nec). The program then calls `nag_dhseqr` (f08pec) to compute the eigenvalues, and `nag_dhsein` (f08pkc) to compute the required eigenvectors of  $H$  by inverse iteration. Finally `nag_dormhr` (f08ngc) is called to transform the eigenvectors of  $H$  back to eigenvectors of the original matrix  $A$ .

### 10.1 Program Text

```
/* nag_dormhr (f08ngc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf08.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, k, m, n, pda, pdh, pdvl, pdvr, pdz;
    Integer tau_len, ifaill_len, select_len, w_len;
    Integer exit_status = 0;
    double thresh;
    Complex eig, eigl;
    /* Arrays */
    double *a = 0, *h = 0, *vl = 0, *vr = 0, *z = 0, *wi = 0, *wr = 0;
    double *tau = 0;
    Integer *ifaill = 0, *ifailr = 0;

    /* Nag Types */
    NagError fail;
    Nag_OrderType order;
    Nag_Boolean *select = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
#define H(I, J) h[(J - 1) * pdh + I - 1]
#define VR(I, J) vr[(J - 1) * pdvr + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
#define H(I, J) h[(I - 1) * pdh + J - 1]
#define VR(I, J) vr[(I - 1) * pdvr + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dormhr (f08ngc) Example Program Results\n\n");

    /* Skip heading in data file */
}
```

```

#ifdef _WIN32
    scanf_s("%*[^\\n] ");
#else
    scanf("%*[^\\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[^\\n] ", &n);
#else
    scanf("%" NAG_IFMT "%*[^\\n] ", &n);
#endif

    pda = n;
    pdh = n;
    pdvl = n;
    pdvr = n;
    pdz = 1;
    tau_len = n;
    w_len = n;
    ifaill_len = n;
    select_len = n;

    /* Allocate memory */
    if (!(a = NAG_ALLOC(n * n, double)) ||
        !(h = NAG_ALLOC(n * n, double)) ||
        !(vl = NAG_ALLOC(n * n, double)) ||
        !(vr = NAG_ALLOC(n * n, double)) ||
        !(z = NAG_ALLOC(1 * 1, double)) ||
        !(wi = NAG_ALLOC(w_len, double)) ||
        !(wr = NAG_ALLOC(w_len, double)) ||
        !(ifaill = NAG_ALLOC(ifaill_len, Integer)) ||
        !(ifailr = NAG_ALLOC(ifaill_len, Integer)) ||
        !(select = NAG_ALLOC(select_len, Nag_Boolean)) ||
        !(tau = NAG_ALLOC(tau_len, double)))
    {
        printf("Allocation failure\\n");
        exit_status = -1;
        goto END;
    }
    /* Read A from data file */
    for (i = 1; i <= n; ++i)
#ifdef _WIN32
        for (j = 1; j <= n; ++j)
            scanf_s("%lf", &A(i, j));
#else
        for (j = 1; j <= n; ++j)
            scanf("%lf", &A(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[^\\n]");
#else
    scanf("%*[^\\n]");
#endif
#ifdef _WIN32
    scanf_s("%lf%*[^\\n]", &thresh);
#else
    scanf("%lf%*[^\\n]", &thresh);
#endif

    /* Reduce A to upper Hessenberg form */
    /* nag_dgehrd (f08nec).
     * Orthogonal reduction of real general matrix to upper
     * Hessenberg form
     */
    nag_dgehrd(order, n, 1, n, a, pda, tau, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_dgehrd (f08nec).\\n%s\\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Copy A to H */

```

```

for (i = 1; i <= n; ++i)
    for (j = 1; j <= n; ++j)
        H(i, j) = A(i, j);

/* Calculate the eigenvalues of H (same as A) */
/* nag_dhseqr (f08pec).
 * Eigenvalues and Schur factorization of real upper
 * Hessenberg matrix reduced from real general matrix
 */
nag_dhseqr(order, Nag_EigVals, Nag_NotZ, n, 1, n, h, pdh, wr,
            wi, z, pdz, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dhseqr (f08pec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print eigenvalues */
printf(" Eigenvalues\n");
for (i = 0; i < n; ++i)
    printf(" (%8.4f,%8.4f)\n", wr[i], wi[i]);
printf("\n");
for (i = 0; i < n; ++i)
    select[i] = wr[i] < thresh ? Nag_TRUE : Nag_FALSE;
/* Calculate the eigenvectors of H (as specified by SELECT), */
/* storing the result in VR */
/* nag_dhsein (f08pkc).
 * Selected right and/or left eigenvectors of real upper
 * Hessenberg matrix by inverse iteration
 */
nag_dhsein(order, Nag_RightSide, Nag_HSEQRSource, Nag_NoVec, select,
            n, a, pda, wr, wi, vl, pdvl, vr, pdvr, n, &m, ifaill,
            ifailr, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dhsein (f08pkc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Calculate the eigenvectors of A = Q * VR */
/* nag_dormhr (f08ngc).
 * Apply orthogonal transformation matrix from reduction to
 * Hessenberg form determined by nag_dgehrd (f08nec)
 */
nag_dormhr(order, Nag_LeftSide, Nag_NoTrans, n, m, 1, n, a, pda,
            tau, vr, pdvr, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dormhr (f08ngc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Scale selected eigenvectors */
j = 0;
for (k = 0; k < n; k++) {
    if (select[k]) {
        j++;
        if (wi[k] == 0.0) {
            for (i = 2; i <= n; i++)
                VR(i, j) = VR(i, j) / VR(1, j);
            VR(1, j) = 1.0;
        }
        else {
            eig1 = nag_complex(VR(1, j), VR(1, j + 1));
            for (i = 1; i <= n; i++) {
                eig = nag_complex(VR(i, j), VR(i, j + 1));
                eig = nag_complex_divide(eig, eig1);
                VR(i, j) = eig.re;
                VR(i, j + 1) = eig.im;
            }
            j++;
        }
    }
}

```

```

        k++;
    }
}
/* Print Eigenvectors */
/* nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
fflush(stdout);
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, m, vr,
    pdvr, "Contents of array VR", 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(a);
NAG_FREE(h);
NAG_FREE(vl);
NAG_FREE(vr);
NAG_FREE(z);
NAG_FREE(wi);
NAG_FREE(wr);
NAG_FREE(ifaill);
NAG_FREE(ifailr);
NAG_FREE(select);
NAG_FREE(tau);
return exit_status;
}

```

## 10.2 Program Data

```

nag_dormhr (f08ngc) Example Program Data
4                               :Value of N
0.35    0.45   -0.14   -0.17
0.09    0.07   -0.54    0.35
-0.44   -0.33   -0.03    0.17
0.25   -0.32   -0.13    0.11   :End of matrix A
0.0                               :Value of THRESH

```

## 10.3 Program Results

nag\_dormhr (f08ngc) Example Program Results

```

Eigenvalues
( 0.7995,  0.0000)
( -0.0994,  0.4008)
( -0.0994, -0.4008)
( -0.1007,  0.0000)

Contents of array VR
      1      2      3
1      1.0000      0.0000      1.0000
2     -1.7779      0.3606      2.6491
3     -0.9521      0.3411      4.7381
4     -1.2785     -1.6841      5.7614

```

---