

# NAG Library Function Document

## nag\_zungbr (f08ktc)

### 1 Purpose

nag\_zungbr (f08ktc) generates one of the complex unitary matrices  $Q$  or  $P^H$  which were determined by nag\_zgebrd (f08ksc) when reducing a complex matrix to bidiagonal form.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_zungbr (Nag_OrderType order, Nag_VectType vect, Integer m,
                 Integer n, Integer k, Complex a[], Integer pda, const Complex tau[],
                 NagError *fail)
```

### 3 Description

nag\_zungbr (f08ktc) is intended to be used after a call to nag\_zgebrd (f08ksc), which reduces a complex rectangular matrix  $A$  to real bidiagonal form  $B$  by a unitary transformation:  $A = QB P^H$ . nag\_zgebrd (f08ksc) represents the matrices  $Q$  and  $P^H$  as products of elementary reflectors.

This function may be used to generate  $Q$  or  $P^H$  explicitly as square matrices, or in some cases just the leading columns of  $Q$  or the leading rows of  $P^H$ .

The various possibilities are specified by the arguments **vect**, **m**, **n** and **k**. The appropriate values to cover the most likely cases are as follows (assuming that  $A$  was an  $m$  by  $n$  matrix):

1. To form the full  $m$  by  $m$  matrix  $Q$ :

```
nag_zungbr (order, Nag_FormQ, m, m, n, ...)
```

(note that the array **a** must have at least  $m$  columns).

2. If  $m > n$ , to form the  $n$  leading columns of  $Q$ :

```
nag_zungbr (order, Nag_FormQ, m, n, n, ...)
```

3. To form the full  $n$  by  $n$  matrix  $P^H$ :

```
nag_zungbr (order, Nag_FormP, n, n, m, ...)
```

(note that the array **a** must have at least  $n$  rows).

4. If  $m < n$ , to form the  $m$  leading rows of  $P^H$ :

```
nag_zungbr (order, Nag_FormP, m, n, m, ...)
```

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType

*Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by

**order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **vect** – Nag\_VectType *Input*

*On entry:* indicates whether the unitary matrix  $Q$  or  $P^H$  is generated.

**vect** = Nag\_FormQ  
 $Q$  is generated.

**vect** = Nag\_FormP  
 $P^H$  is generated.

*Constraint:* **vect** = Nag\_FormQ or Nag\_FormP.

3: **m** – Integer *Input*

*On entry:*  $m$ , the number of rows of the unitary matrix  $Q$  or  $P^H$  to be returned.

*Constraint:* **m**  $\geq 0$ .

4: **n** – Integer *Input*

*On entry:*  $n$ , the number of columns of the unitary matrix  $Q$  or  $P^H$  to be returned.

*Constraints:*

**n**  $\geq 0$ ;  
 if **vect** = Nag\_FormQ and **m**  $>$  **k**, **m**  $\geq$  **n**  $\geq$  **k**;  
 if **vect** = Nag\_FormQ and **m**  $\leq$  **k**, **m** = **n**;  
 if **vect** = Nag\_FormP and **n**  $>$  **k**, **n**  $\geq$  **m**  $\geq$  **k**;  
 if **vect** = Nag\_FormP and **n**  $\leq$  **k**, **n** = **m**.

5: **k** – Integer *Input*

*On entry:* if **vect** = Nag\_FormQ, the number of columns in the original matrix  $A$ .

If **vect** = Nag\_FormP, the number of rows in the original matrix  $A$ .

*Constraint:* **k**  $\geq 0$ .

6: **a**[*dim*] – Complex *Input/Output*

**Note:** the dimension, *dim*, of the array **a** must be at least

$\max(1, \mathbf{pda} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pda})$  when **order** = Nag\_RowMajor.

*On entry:* details of the vectors which define the elementary reflectors, as returned by nag\_zgebrd (f08ksc).

*On exit:* the unitary matrix  $Q$  or  $P^H$ , or the leading rows or columns thereof, as specified by **vect**, **m** and **n**.

If **order** = Nag\_ColMajor, the  $(i, j)$ th element of the matrix is stored in **a**[( $j - 1$ )  $\times$  **pda** +  $i - 1$ ].

If **order** = Nag\_RowMajor, the  $(i, j)$ th element of the matrix is stored in **a**[( $i - 1$ )  $\times$  **pda** +  $j - 1$ ].

7: **pda** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **a**.

*Constraint:* **pda**  $\geq \max(1, \mathbf{m})$ .

8: **tau**[*dim*] – const Complex

*Input*

**Note:** the dimension, *dim*, of the array **tau** must be at least

$\max(1, \min(\mathbf{m}, \mathbf{k}))$  when **vect** = Nag\_FormQ;  
 $\max(1, \min(\mathbf{n}, \mathbf{k}))$  when **vect** = Nag\_FormP.

*On entry:* further details of the elementary reflectors, as returned by nag\_zgebrd (f08ksc) in its argument **tauq** if **vect** = Nag\_FormQ, or in its argument **taup** if **vect** = Nag\_FormP.

9: **fail** – NagError \*

*Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument *⟨value⟩* had an illegal value.

### NE\_ENUM\_INT\_3

On entry, **vect** = *⟨value⟩*, **m** = *⟨value⟩*, **n** = *⟨value⟩* and **k** = *⟨value⟩*.

Constraint:  $\mathbf{n} \geq 0$  and

if **vect** = Nag\_FormQ and  $\mathbf{m} > \mathbf{k}$ ,  $\mathbf{m} \geq \mathbf{n} \geq \mathbf{k}$ ;

if **vect** = Nag\_FormQ and  $\mathbf{m} \leq \mathbf{k}$ ,  $\mathbf{m} = \mathbf{n}$ ;

if **vect** = Nag\_FormP and  $\mathbf{n} > \mathbf{k}$ ,  $\mathbf{n} \geq \mathbf{m} \geq \mathbf{k}$ ;

if **vect** = Nag\_FormP and  $\mathbf{n} \leq \mathbf{k}$ ,  $\mathbf{n} = \mathbf{m}$ .

### NE\_INT

On entry, **k** = *⟨value⟩*.

Constraint:  $\mathbf{k} \geq 0$ .

On entry, **m** = *⟨value⟩*.

Constraint:  $\mathbf{m} \geq 0$ .

On entry, **pda** = *⟨value⟩*.

Constraint: **pda** > 0.

### NE\_INT\_2

On entry, **pda** = *⟨value⟩* and **m** = *⟨value⟩*.

Constraint: **pda**  $\geq \max(1, \mathbf{m})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed matrix  $Q$  differs from an exactly unitary matrix by a matrix  $E$  such that

$$\|E\|_2 = O(\epsilon),$$

where  $\epsilon$  is the *machine precision*. A similar statement holds for the computed matrix  $P^H$ .

## 8 Parallelism and Performance

nag\_zungbr (f08ktc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_zungbr (f08ktc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of real floating-point operations for the cases listed in Section 3 are approximately as follows:

1. To form the whole of  $Q$ :

$$\begin{aligned} & \frac{16}{3}n(3m^2 - 3mn + n^2) \text{ if } m > n, \\ & \frac{16}{3}m^3 \text{ if } m \leq n; \end{aligned}$$

2. To form the  $n$  leading columns of  $Q$  when  $m > n$ :

$$\frac{8}{3}n^2(3m - n);$$

3. To form the whole of  $P^H$ :

$$\begin{aligned} & \frac{16}{3}n^3 \text{ if } m \geq n, \\ & \frac{16}{3}m^3(3n^2 - 3mn + m^2) \text{ if } m < n; \end{aligned}$$

4. To form the  $m$  leading rows of  $P^H$  when  $m < n$ :

$$\frac{8}{3}m^2(3n - m).$$

The real analogue of this function is nag\_dorgbr (f08kfc).

## 10 Example

For this function two examples are presented, both of which involve computing the singular value decomposition of a matrix  $A$ , where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ -0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}$$

in the first example and

$$A = \begin{pmatrix} 0.28 - 0.36i & 0.50 - 0.86i & -0.77 - 0.48i & 1.58 + 0.66i \\ -0.50 - 1.10i & -1.21 + 0.76i & -0.32 - 0.24i & -0.27 - 1.15i \\ 0.36 - 0.51i & -0.07 + 1.33i & -0.75 + 0.47i & -0.08 + 1.01i \end{pmatrix}$$

in the second.  $A$  must first be reduced to tridiagonal form by `nag_zgebrd` (f08ksc). The program then calls `nag_zungbr` (f08ktc) twice to form  $Q$  and  $P^H$ , and passes these matrices to `nag_zbdsqr` (f08msc), which computes the singular value decomposition of  $A$ .

## 10.1 Program Text

```
/* nag_zungbr (f08ktc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx04.h>
int main(void)
{
    /* Scalars */
    Integer i, ic, j, m, n, pda, pdc, pdu, pdvt, d_len;
    Integer e_len, tauq_len, taup_len;
    Integer exit_status = 0;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    Complex *a = 0, *c = 0, *taup = 0, *tauq = 0, *u = 0, *vt = 0;
    double *d = 0, *e = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J)  a[(J-1)*pda + I - 1]
#define VT(I, J) vt[(J-1)*pdvt + I - 1]
#define U(I, J)  u[(J-1)*pdu + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J)  a[(I-1)*pda + J - 1]
#define VT(I, J) vt[(I-1)*pdvt + J - 1]
#define U(I, J)  u[(I-1)*pdu + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zungbr (f08ktc) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    for (ic = 1; ic <= 2; ++ic) {
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m, &n);
#else
        scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m, &n);
#endif
    }
    d_len = n;
#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdc = n;
    pdu = m;
    pdvt = m;
    e_len = n - 1;
    tauq_len = n;

```

```

    taup_len = n;
#else
    pda = n;
    pdc = n;
    pdu = n;
    pdvt = n;
    e_len = n - 1;
    tauq_len = n;
    taup_len = n;
#endif
/* Allocate memory */
if (!(a = NAG_ALLOC(m * n, Complex)) ||
    !(c = NAG_ALLOC(n * n, Complex)) ||
    !(taup = NAG_ALLOC(taup_len, Complex)) ||
    !(tauq = NAG_ALLOC(tauq_len, Complex)) ||
    !(u = NAG_ALLOC(m * n, Complex)) ||
    !(vt = NAG_ALLOC(m * n, Complex)) ||
    !(d = NAG_ALLOC(d_len, double)) || !(e = NAG_ALLOC(e_len, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto ENDL;
}
/* Read A from data file */
for (i = 1; i <= m; ++i) {
    for (j = 1; j <= n; ++j)
#ifdef _WIN32
        scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
        scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
}
#ifdef _WIN32
    scanf_s("%*[^\\n] ");
#else
    scanf("%*[^\\n] ");
#endif
/* Reduce A to bidiagonal form */
/* nag_zgebrd (f08ksc).
 * Unitary reduction of complex general rectangular matrix
 * to bidiagonal form
 */
nag_zgebrd(order, m, n, a, pda, d, e, tauq, taup, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgebrd (f08ksc).\\n%s\\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
if (m >= n) {
    /* Copy A to VT and U */
    for (i = 1; i <= n; ++i) {
        for (j = i; j <= n; ++j) {
            VT(i, j).re = A(i, j).re;
            VT(i, j).im = A(i, j).im;
        }
    }
    for (i = 1; i <= m; ++i) {
        for (j = 1; j <= MIN(i, n); ++j) {
            U(i, j).re = A(i, j).re;
            U(i, j).im = A(i, j).im;
        }
    }
    /* Form P^H explicitly, storing the result in VT */
    /* nag_zungbr (f08ktc).
     * Generate unitary transformation matrices from reduction
     * to bidiagonal form determined by nag_zgebrd (f08ksc)
     */
    nag_zungbr(order, Nag_FormP, n, n, m, vt, pdvt, taup, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zungbr (f08ktc).\\n%s\\n", fail.message);
        exit_status = 1;
    }
}

```

```

    goto ENDL;
}

/* Form Q explicitly, storing the result in U */
/* nag_zungbr (f08ktc), see above. */
nag_zungbr(order, Nag_FormQ, m, n, n, u, pdu, tauq, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zungbr (f08ktc).\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}

/* Compute the SVD of A */
/* nag_zbdsqr (f08msc).
 * SVD of real bidiagonal matrix reduced from complex
 * general matrix
 */
nag_zbdsqr(order, Nag_Upper, n, n, m, 0, d, e, vt, pdvt, u,
           pdu, c, pdc, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zbdsqr (f08msc).\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}

/* Print singular values, left & right singular vectors */
printf("\nExample 1: singular values\n");
for (i = 1; i <= n; ++i)
    printf("%8.4f%s", d[i - 1], i % 8 == 0 ? "\n": " ");
printf("\n\n");
/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print complex general matrix (comprehensive)
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix,
                             Nag_NonUnitDiag, n, n, vt, pdvt,
                             Nag_BracketForm, "%7.4f",
                             "Example 1: right singular vectors, "
                             "by row", Nag_IntegerLabels, 0,
                             Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc). "
           "\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
printf("\n");
/* nag_gen_complx_mat_print_comp (x04dbc), see above. */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix,
                             Nag_NonUnitDiag, m, n, u, pdu,
                             Nag_BracketForm, "%7.4f",
                             "Example 1: left singular vectors, "
                             "by column", Nag_IntegerLabels, 0,
                             Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc). "
           "\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
}
else {
    /* Copy A to VT and U */
    for (i = 1; i <= m; ++i) {
        for (j = i; j <= n; ++j) {
            VT(i, j).re = A(i, j).re;
            VT(i, j).im = A(i, j).im;
        }
    }
    for (i = 1; i <= m; ++i) {
        for (j = 1; j <= i; ++j) {

```

```

        U(i, j).re = A(i, j).re;
        U(i, j).im = A(i, j).im;
    }
}
/* Form P^H explicitly, storing the result in VT */
/* nag_zungbr (f08ktc), see above. */
nag_zungbr(order, Nag_FormP, m, n, m, vt, pdvt, tau_p, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zungbr (f08ktc).\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
/* Form Q explicitly, storing the result in U */
/* nag_zungbr (f08ktc), see above. */
nag_zungbr(order, Nag_FormQ, m, m, n, u, pdu, tau_q, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zungbr (f08ktc).\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
/* Compute the SVD of A */
/* nag_zbdsqr (f08msc), see above. */
nag_zbdsqr(order, Nag_Lower, m, n, m, 0, d, e, vt, pdvt, u,
           pdu, c, pdc, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zbdsqr (f08msc).\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
/* Print singular values, left & right singular vectors */
printf("\nExample 2: singular values\n");
for (i = 1; i <= m; ++i)
    printf("%8.4f%s", d[i - 1], i % 8 == 0 ? "\n" : " ");
printf("\n\n");
/* nag_gen_complx_mat_print_comp (x04dbc), see above. */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix,
                             Nag_NonUnitDiag, m, n, vt, pdvt,
                             Nag_BracketForm, "%7.4f",
                             "Example 2: right singular vectors, "
                             "by row", Nag_IntegerLabels, 0,
                             Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc). "
           "\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
printf("\n");
/* nag_gen_complx_mat_print_comp (x04dbc), see above. */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix,
                             Nag_NonUnitDiag, m, m, u, pdu,
                             Nag_BracketForm, "%7.4f",
                             "Example 2: left singular vectors, "
                             "by column", Nag_IntegerLabels, 0,
                             Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc). "
           "\n%s\n", fail.message);
    exit_status = 1;
    goto ENDL;
}
}
ENDL:
NAG_FREE(a);
NAG_FREE(c);
NAG_FREE(tau_p);
NAG_FREE(tau_q);
NAG_FREE(u);
NAG_FREE(vt);

```



```

    NAG_FREE(d);
    NAG_FREE(e);
}
NAG_FREE(a);
NAG_FREE(c);
NAG_FREE(taup);
NAG_FREE(tauq);
NAG_FREE(u);
NAG_FREE(vt);
NAG_FREE(d);
NAG_FREE(e);

return exit_status;
}

```

## 10.2 Program Data

nag\_zungbr (f08ktc) Example Program Data

```

6 4                                     :Values of M and N, Example 1
( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
(-0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26) :End of matrix A
3 4                                     :Values of M and N, Example 2
( 0.28,-0.36) ( 0.50,-0.86) (-0.77,-0.48) ( 1.58, 0.66)
(-0.50,-1.10) (-1.21, 0.76) (-0.32,-0.24) (-0.27,-1.15)
( 0.36,-0.51) (-0.07, 1.33) (-0.75, 0.47) (-0.08, 1.01) :End of matrix A

```

## 10.3 Program Results

nag\_zungbr (f08ktc) Example Program Results

Example 1: singular values

```
3.9994  3.0003  1.9944  0.9995
```

Example 1: right singular vectors, by row

```

1 2 3 4
1 (-0.6971,-0.0000) (-0.0867,-0.3548) ( 0.0560,-0.5400) (-0.1878,-0.2253)
2 ( 0.2403, 0.0000) ( 0.0725,-0.2336) (-0.2477,-0.5291) ( 0.7026, 0.2177)
3 (-0.5123, 0.0000) (-0.3030,-0.1735) ( 0.0678, 0.5162) ( 0.4418, 0.3864)
4 (-0.4403, 0.0000) ( 0.5294, 0.6361) (-0.3027,-0.0346) ( 0.1667, 0.0258)

```

Example 1: left singular vectors, by column

```

1 2 3 4
1 (-0.5634, 0.0016) (-0.2687,-0.2749) ( 0.2451, 0.4657) ( 0.3787, 0.2987)
2 ( 0.1205,-0.6108) (-0.2909, 0.1085) ( 0.4329,-0.1758) (-0.0182,-0.0437)
3 (-0.0816, 0.1613) (-0.1660, 0.3885) (-0.4667, 0.3821) (-0.0800,-0.2276)
4 ( 0.1441,-0.1532) ( 0.1984,-0.1737) (-0.0034, 0.1555) ( 0.2608,-0.5382)
5 (-0.2487,-0.0926) ( 0.6253, 0.3304) ( 0.2643,-0.0194) ( 0.1002, 0.0140)
6 (-0.3758, 0.0793) (-0.0307,-0.0816) ( 0.1266, 0.1747) (-0.4175,-0.4058)

```

Example 2: singular values

```
3.0004  1.9967  0.9973
```

Example 2: right singular vectors, by row

```

1 2 3 4
1 ( 0.2454,-0.0001) ( 0.2942,-0.5843) ( 0.0162,-0.0810) ( 0.6794, 0.2083)
2 (-0.1692, 0.5194) ( 0.1915,-0.4374) ( 0.5205,-0.0244) (-0.3149,-0.3208)
3 (-0.5553, 0.1403) ( 0.1438,-0.1507) (-0.5684,-0.5505) (-0.0318,-0.0378)

```

Example 2: left singular vectors, by column

```

1 2 3
1 ( 0.6518, 0.0000) (-0.4312, 0.0000) ( 0.6239, 0.0000)
2 (-0.4437,-0.5027) (-0.3794, 0.1026) ( 0.2014, 0.5961)
3 (-0.2012, 0.2916) (-0.8122, 0.0030) (-0.3511,-0.3026)

```

---