

# NAG Library Function Document

## nag\_zpftri (f07wwc)

### 1 Purpose

nag\_zpftri (f07wwc) computes the inverse of a complex Hermitian positive definite matrix using the Cholesky factorization computed by nag\_zpftrf (f07wrc) stored in Rectangular Full Packed (RFP) format.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zpftri (Nag_OrderType order, Nag_RFP_Store transr,
                 Nag_UploType uplo, Integer n, Complex ar[], NagError *fail)
```

### 3 Description

nag\_zpftri (f07wwc) is used to compute the inverse of a complex Hermitian positive definite matrix  $A$ , stored in RFP format. The RFP storage format is described in Section 3.3.3 in the f07 Chapter Introduction. The function must be preceded by a call to nag\_zpftrf (f07wrc), which computes the Cholesky factorization of  $A$ .

If **uplo** = Nag\_Upper,  $A = U^H U$  and  $A^{-1}$  is computed by first inverting  $U$  and then forming  $(U^{-1})U^{-H}$ .

If **uplo** = Nag\_Lower,  $A = L L^H$  and  $A^{-1}$  is computed by first inverting  $L$  and then forming  $L^{-H}(L^{-1})$ .

### 4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

Gustavson F G, Waśniewski J, Dongarra J J and Langou J (2010) Rectangular full packed format for Cholesky's algorithm: factorization, solution, and inversion *ACM Trans. Math. Software* **37**, 2

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **transr** – Nag\_RFP\_Store *Input*

*On entry:* specifies whether the normal RFP representation of  $A$  or its conjugate transpose is stored.

**transr** = Nag\_RFP\_Normal

The matrix  $A$  is stored in normal RFP format.

**transr** = Nag\_RFP\_ConjTrans

The conjugate transpose of the RFP representation of the matrix  $A$  is stored.

*Constraint:* **transr** = Nag\_RFP\_Normal or Nag\_RFP\_ConjTrans.

- 3: **uplo** – Nag\_UploType *Input*  
*On entry:* specifies how  $A$  has been factorized.  
**uplo** = Nag\_Upper  
 $A = U^H U$ , where  $U$  is upper triangular.  
**uplo** = Nag\_Lower  
 $A = L L^H$ , where  $L$  is lower triangular.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 4: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 5: **ar**[ $n \times (n + 1)/2$ ] – Complex *Input/Output*  
*On entry:* the Cholesky factorization of  $A$  stored in RFP format, as returned by nag\_zpfrf (f07wrc).  
*On exit:* the factorization is overwritten by the  $n$  by  $n$  matrix  $A^{-1}$  stored in RFP format.
- 6: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $n = \langle value \rangle$ .

Constraint:  $n \geq 0$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_MAT\_NOT\_POS\_DEF

The leading minor of order  $\langle value \rangle$  is not positive definite and the factorization could not be completed. Hence  $A$  itself is not positive definite. This may indicate an error in forming the matrix  $A$ . There is no function specifically designed to invert a Hermitian matrix stored in RFP format which is not positive definite; the matrix must be treated as a full Hermitian matrix, by calling nag\_zhetri (f07mwc).

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

The computed inverse  $X$  satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where  $c(n)$  is a modest function of  $n$ ,  $\epsilon$  is the *machine precision* and  $\kappa_2(A)$  is the condition number of  $A$  defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

**8 Parallelism and Performance**

nag\_zpftri (f07wwc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The total number of real floating-point operations is approximately  $\frac{8}{3}n^3$ .

The real analogue of this function is nag\_dpfttri (f07wjc).

**10 Example**

This example computes the inverse of the matrix  $A$ , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian positive definite, stored in RFP format, and must first be factorized by nag\_zpftfr (f07wrc).

**10.1 Program Text**

```
/* nag_zpftri (f07wwc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf01.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
```

```

Integer exit_status = 0;
Integer i, j, k, lar1, lar2, lenar, n, pdar, pda, q;
/* Arrays */
Complex *ar = 0, *a = 0;
char nag_enum_arg[40];
/* NAG types */
Nag_RFP_Store transr;
Nag_UploType uplo;
Nag_OrderType order;
Nag_MatrixType matrix;
Nag_DiagType diag = Nag_NonUnitDiag;
NagError fail;

#ifdef NAG_COLUMN_MAJOR
    order = Nag_ColMajor;
#define AR(I,J) ar[J*pdar + I]
#else
    order = Nag_RowMajor;
#define AR(I,J) ar[I*pdar + J]
#endif

    INIT_FAIL(fail);
    printf("nag_zpftri (f07wwc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &n);
#else
    scanf("%" NAG_IFMT "", &n);
#endif
#ifdef _WIN32
    scanf_s("%39s", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s", nag_enum_arg);
#endif
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s("%39s*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s*[\n]", nag_enum_arg);
#endif
    transr = (Nag_RFP_Store) nag_enum_name_to_value(nag_enum_arg);

    lenar = (n * (n + 1)) / 2;
    if (!(ar = NAG_ALLOC(lenar, Complex)) || !(a = NAG_ALLOC(n * n, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    pda = n;

    /* Setup dimensions for RFP array ar. */
    k = n / 2;
    q = n - k;
    if (transr == Nag_RFP_Normal) {
        lar1 = 2 * k + 1;
        lar2 = q;
    }
    else {
        lar1 = q;
        lar2 = 2 * k + 1;
    }
    if (order == Nag_RowMajor) {
        pdar = lar2;
    }
    else {

```

```

    pdar = lar1;
}
/* Read matrix into RFP array ar. */
for (i = 0; i < lar1; i++) {
    for (j = 0; j < lar2; j++) {
#ifdef _WIN32
        scanf_s(" ( %lf , %lf ) ", &AR(i, j).re, &AR(i, j).im);
#else
        scanf(" ( %lf , %lf ) ", &AR(i, j).re, &AR(i, j).im);
#endif
    }
}

/* Factorize A using nag_zpftrf (f07wrc) which performs a Cholesky
 * factorization of a complex Hermitian positive definite matrix in
 * Rectangular Full Packed format
 */
nag_zpftrf(order, transr, uplo, n, ar, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zpftrf (f07wrc)\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute inverse of A using nag_zpftri (f07wwc). */
nag_zpftri(order, transr, uplo, n, ar, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zpftri (f07wwc)\n%s\n", fail.message);
    exit_status = 2;
    goto END;
}

/* Convert A to full matrix format using nag_ztfttr (f01vhc). */
nag_ztfttr(order, transr, uplo, n, ar, a, pda, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ztfttr (f01vhc)\n%s\n", fail.message);
    exit_status = 3;
    goto END;
}

matrix = (uplo == Nag_Lower ? Nag_LowerMatrix : Nag_UpperMatrix);

fflush(stdout);
/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print complex general matrix (comprehensive).
 */
nag_gen_complx_mat_print_comp(order, matrix, diag, n, n, a, pda,
                              Nag_BracketForm, "%7.4f", "Inverse",
                              Nag_IntegerLabels, 0, Nag_IntegerLabels,
                              0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc)\n%s\n",
          fail.message);
    exit_status = 4;
}

END:
NAG_FREE(ar);
NAG_FREE(a);
return exit_status;
}

```

## 10.2 Program Data

```
nag_zpftri (f07wwc) Example Program Data
  4   Nag_Lower  Nag_RFP_Normal      : n, uplo, transr

( 4.09, 0.00)   ( 2.33,-0.14)
( 3.23, 0.00)   ( 4.29, 0.00)
( 1.51, 1.92)   ( 3.58, 0.00)
( 1.90,-0.84)   (-0.23,-1.11)
( 0.42,-2.50)   (-1.18,-1.37)      : ar[]
```

## 10.3 Program Results

nag\_zpftri (f07wwc) Example Program Results

Inverse	1	2	3	4
1	( 5.4691, 0.0000)			
2	(-1.2624,-1.5491)	( 1.1024, 0.0000)		
3	(-2.9746,-0.9616)	( 0.8989,-0.5672)	( 2.1589,-0.0000)	
4	( 1.1962, 2.9772)	(-0.9826,-0.2566)	(-1.3756,-1.4550)	( 2.2934, 0.0000)

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