

# NAG Library Function Document

## nag\_dtbrfs (f07vhc)

### 1 Purpose

nag\_dtbrfs (f07vhc) returns error bounds for the solution of a real triangular band system of linear equations with multiple right-hand sides,  $AX = B$  or  $A^T X = B$ .

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dtbrfs (Nag_OrderType order, Nag_UploType uplo,
                 Nag_TransType trans, Nag_DiagType diag, Integer n, Integer kd,
                 Integer nrhs, const double ab[], Integer pdab, const double b[],
                 Integer pdb, const double x[], Integer pdx, double ferr[],
                 double berr[], NagError *fail)
```

### 3 Description

nag\_dtbrfs (f07vhc) returns the backward errors and estimated bounds on the forward errors for the solution of a real triangular band system of linear equations with multiple right-hand sides  $AX = B$  or  $A^T X = B$ . The function handles each right-hand side vector (stored as a column of the matrix  $B$ ) independently, so we describe the function of nag\_dtbrfs (f07vhc) in terms of a single right-hand side  $b$  and solution  $x$ .

Given a computed solution  $x$ , the function computes the *component-wise backward error*  $\beta$ . This is the size of the smallest relative perturbation in each element of  $A$  and  $b$  such that  $x$  is the exact solution of a perturbed system

$$| \delta a_{ij} | \leq \beta | a_{ij} | \quad \text{and} \quad | \delta b_i | \leq \beta | b_i |. \quad (A + \delta A)x = b + \delta b$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where  $\hat{x}$  is the true solution.

For details of the method, see the f07 Chapter Introduction.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

- 2:     **uplo** – Nag\_UploType *Input*  
*On entry:* specifies whether  $A$  is upper or lower triangular.  
**uplo** = Nag\_Upper  
 $A$  is upper triangular.  
**uplo** = Nag\_Lower  
 $A$  is lower triangular.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 3:     **trans** – Nag\_TransType *Input*  
*On entry:* indicates the form of the equations.  
**trans** = Nag\_NoTrans  
The equations are of the form  $AX = B$ .  
**trans** = Nag\_Trans or Nag\_ConjTrans  
The equations are of the form  $A^T X = B$ .  
*Constraint:* **trans** = Nag\_NoTrans, Nag\_Trans or Nag\_ConjTrans.
- 4:     **diag** – Nag\_DiagType *Input*  
*On entry:* indicates whether  $A$  is a nonunit or unit triangular matrix.  
**diag** = Nag\_NonUnitDiag  
 $A$  is a nonunit triangular matrix.  
**diag** = Nag\_UnitDiag  
 $A$  is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.  
*Constraint:* **diag** = Nag\_NonUnitDiag or Nag\_UnitDiag.
- 5:     **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:* **n**  $\geq 0$ .
- 6:     **kd** – Integer *Input*  
*On entry:*  $k_d$ , the number of superdiagonals of the matrix  $A$  if **uplo** = Nag\_Upper, or the number of subdiagonals if **uplo** = Nag\_Lower.  
*Constraint:* **kd**  $\geq 0$ .
- 7:     **nrhs** – Integer *Input*  
*On entry:*  $r$ , the number of right-hand sides.  
*Constraint:* **nrhs**  $\geq 0$ .
- 8:     **ab**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **ab** must be at least  $\max(1, \mathbf{pdab} \times \mathbf{n})$ .  
*On entry:* the  $n$  by  $n$  triangular band matrix  $A$ .  
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of  $A_{ij}$ , depends on the **order** and **uplo** arguments as follows:

if **order** = Nag\_ColMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab** $[k_d + i - j + (j - 1) \times \mathbf{pdab}]$ , for  $j = 1, \dots, n$  and  
 $i = \max(1, j - k_d), \dots, j$ ;  
 if **order** = Nag\_ColMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab** $[i - j + (j - 1) \times \mathbf{pdab}]$ , for  $j = 1, \dots, n$  and  
 $i = j, \dots, \min(n, j + k_d)$ ;  
 if **order** = Nag\_RowMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab** $[j - i + (i - 1) \times \mathbf{pdab}]$ , for  $i = 1, \dots, n$  and  
 $j = i, \dots, \min(n, i + k_d)$ ;  
 if **order** = Nag\_RowMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab** $[k_d + j - i + (i - 1) \times \mathbf{pdab}]$ , for  $i = 1, \dots, n$  and  
 $j = \max(1, i - k_d), \dots, i$ .

If **diag** = Nag\_UnitDiag, the diagonal elements of AB are assumed to be 1, and are not referenced.

9: **pdab** – Integer Input

*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **ab**.

*Constraint:* **pdab**  $\geq$  **kd** + 1.

10: **b** $[dim]$  – const double Input

**Note:** the dimension,  $dim$ , of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $B$  is stored in

**b** $[(j - 1) \times \mathbf{pdb} + i - 1]$  when **order** = Nag\_ColMajor;  
**b** $[(i - 1) \times \mathbf{pdb} + j - 1]$  when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .

11: **pdb** – Integer Input

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.

*Constraints:*

if **order** = Nag\_ColMajor, **pdb**  $\geq$   $\max(1, \mathbf{n})$ ;  
 if **order** = Nag\_RowMajor, **pdb**  $\geq$   $\max(1, \mathbf{nrhs})$ .

12: **x** $[dim]$  – const double Input

**Note:** the dimension,  $dim$ , of the array **x** must be at least

$\max(1, \mathbf{pdx} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdx})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $X$  is stored in

**x** $[(j - 1) \times \mathbf{pdx} + i - 1]$  when **order** = Nag\_ColMajor;  
**x** $[(i - 1) \times \mathbf{pdx} + j - 1]$  when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $r$  solution matrix  $X$ , as returned by nag\_dtbtrs (f07vec).

13: **pdx** – Integer Input

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **x**.

*Constraints:*

if **order** = Nag\_ColMajor, **pdx**  $\geq \max(1, \mathbf{n})$ ;  
 if **order** = Nag\_RowMajor, **pdx**  $\geq \max(1, \mathbf{nrhs})$ .

14: **ferr**[**nrhs**] – double *Output*

*On exit:* **ferr**[ $j - 1$ ] contains an estimated error bound for the  $j$ th solution vector, that is, the  $j$ th column of  $X$ , for  $j = 1, 2, \dots, r$ .

15: **berr**[**nrhs**] – double *Output*

*On exit:* **berr**[ $j - 1$ ] contains the component-wise backward error bound  $\beta$  for the  $j$ th solution vector, that is, the  $j$ th column of  $X$ , for  $j = 1, 2, \dots, r$ .

16: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **kd** =  $\langle value \rangle$ .

Constraint: **kd**  $\geq 0$ .

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **nrhs** =  $\langle value \rangle$ .

Constraint: **nrhs**  $\geq 0$ .

On entry, **pdab** =  $\langle value \rangle$ .

Constraint: **pdab**  $> 0$ .

On entry, **pdb** =  $\langle value \rangle$ .

Constraint: **pdb**  $> 0$ .

On entry, **pdx** =  $\langle value \rangle$ .

Constraint: **pdx**  $> 0$ .

### NE\_INT\_2

On entry, **pdab** =  $\langle value \rangle$  and **kd** =  $\langle value \rangle$ .

Constraint: **pdab**  $\geq \mathbf{kd} + 1$ .

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq \max(1, \mathbf{nrhs})$ .

On entry, **pdx** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **pdx**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdx** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .  
 Constraint: **pdx**  $\geq \max(1, \mathbf{nrhs})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The bounds returned in **fer** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

## 8 Parallelism and Performance

nag\_dtbrfs (f07vhc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dtbrfs (f07vhc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

A call to nag\_dtbrfs (f07vhc), for each right-hand side, involves solving a number of systems of linear equations of the form  $Ax = b$  or  $A^T x = b$ ; the number is usually 4 or 5 and never more than 11. Each solution involves approximately  $2nk$  floating-point operations (assuming  $n \gg k$ ).

The complex analogue of this function is nag\_ztbrfs (f07vvc).

## 10 Example

This example solves the system of equations  $AX = B$  and to compute forward and backward error bounds, where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -16.64 & -4.16 \\ -13.78 & -16.59 \\ 13.10 & -4.94 \\ -14.14 & -9.96 \end{pmatrix}.$$

### 10.1 Program Text

```
/* nag_dtbrfs (f07vhc) Example Program.
*
* NAGPRODCODE Version.
*
* Copyright 2016 Numerical Algorithms Group.
*
* Mark 26, 2016.
*/
```

```

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, k, kd, n, nrhs, pdab, pdb, pdx;
    Integer ferr_len, berr_len;
    Integer exit_status = 0;
    Nag_UploType uplo;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    char nag_enum_arg[40];
    double *ab = 0, *b = 0, *berr = 0, *ferr = 0, *x = 0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I, J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
#define B(I, J) b[(J-1)*pdb + I - 1]
#define X(I, J) x[(J-1)*pdx + I - 1]
    order = Nag_ColMajor;
#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + k + J - I - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
#define X(I, J) x[(I-1)*pdx + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dtbrfs (f07vhc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kd, &nrhs);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kd, &nrhs);
#endif
    pdab = kd + 1;
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
    pdx = n;
#else
    pdb = nrhs;
    pdx = nrhs;
#endif

    ferr_len = nrhs;
    berr_len = nrhs;

    /* Allocate memory */
    if (!(berr = NAG_ALLOC(berr_len, double)) ||
        !(ferr = NAG_ALLOC(ferr_len, double)) ||
        !(ab = NAG_ALLOC((kd + 1) * n, double)) ||
        !(b = NAG_ALLOC(n * nrhs, double)) ||
        !(x = NAG_ALLOC(n * nrhs, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}

```

```

/* Read A and B from data file, and copy B to X */
#ifdef _WIN32
    scanf_s(" %39s%[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%[\n] ", nag_enum_arg);
#endif
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

k = kd + 1;
if (uplo == Nag_Upper) {
    for (i = 1; i <= n; ++i) {
        for (j = i; j <= MIN(i + kd, n); ++j)
#ifdef _WIN32
            scanf_s("%lf", &AB_UPPER(i, j));
#else
            scanf("%lf", &AB_UPPER(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
} else {
    for (i = 1; i <= n; ++i) {
        for (j = MAX(1, i - kd); j <= i; ++j)
#ifdef _WIN32
            scanf_s("%lf", &AB_LOWER(i, j));
#else
            scanf("%lf", &AB_LOWER(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

for (i = 1; i <= n; ++i) {
    for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
        scanf_s("%lf", &B(i, j));
#else
        scanf("%lf", &B(i, j));
#endif
}
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
/* Copy B to X */
for (i = 1; i <= n; ++i) {
    for (j = 1; j <= nrhs; ++j)
        X(i, j) = B(i, j);
}
/* Compute solution in the array X */
/* nag_dtbtrs (f07vec).
 * Solution of real band triangular system of linear
 * equations, multiple right-hand sides
 */
nag_dtbtrs(order, uplo, Nag_NoTrans, Nag_NonUnitDiag, n,
           kd, nrhs, ab, pdab, x, pdx, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dtbtrs (f07vec).\n%s\n", fail.message);
}

```

```

    exit_status = 1;
    goto END;
}
/* Improve solution, and compute backward errors and */
/* estimated bounds on the forward errors */
/* nag_dtbrfs (f07vhc).
 * Error bounds for solution of real band triangular system
 * of linear equations, multiple right-hand sides
 */
nag_dtbrfs(order, uplo, Nag_NoTrans, Nag_NonUnitDiag, n,
           kd, nrhs, ab, pdab, b, pdb, x, pdx, ferr, berr, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dtbrfs (f07vhc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print details of solution */

/* nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
fflush(stdout);
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs,
                      x, pdx, "Solution(s)", 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
printf("\nBackward errors (machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    printf("%11.1e%s", berr[j - 1], j % 7 == 0 ? "\n" : " ");
printf("\nEstimated forward error bounds " "(machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    printf("%11.1e%s", ferr[j - 1], j % 7 == 0 ? "\n" : " ");
printf("\n");
END:
    NAG_FREE(berr);
    NAG_FREE(ferr);
    NAG_FREE(ab);
    NAG_FREE(b);
    NAG_FREE(x);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_dtbrfs (f07vhc) Example Program Data
  4  1  2                               :Values of n, kd and nrhs
  Nag_Lower                             :Value of uplo
-4.16
-2.25   4.78
        5.86   6.32
      -4.82   0.16   :End of matrix A
-16.64  -4.16
-13.78 -16.59
  13.10  -4.94
-14.14  -9.96       :End of matrix B

```

## 10.3 Program Results

nag\_dtbrfs (f07vhc) Example Program Results

```

Solution(s)
      1      2
1      4.0000   1.0000
2     -1.0000  -3.0000
3      3.0000   2.0000
4      2.0000  -2.0000

```

Backward errors (machine-dependent)  
4.7e-17 2.5e-17  
Estimated forward error bounds (machine-dependent)  
5.4e-14 5.8e-14

---