

# NAG Library Function Document

## nag\_dpbequ (f07hfc)

### 1 Purpose

nag\_dpbequ (f07hfc) computes a diagonal scaling matrix  $S$  intended to equilibrate a real  $n$  by  $n$  symmetric positive definite band matrix  $A$ , with bandwidth  $(2k_d + 1)$ , and reduce its condition number.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dpbequ (Nag_OrderType order, Nag_UploType uplo, Integer n,
                 Integer kd, const double ab[], Integer pdab, double s[], double *scond,
                 double *amax, NagError *fail)
```

### 3 Description

nag\_dpbequ (f07hfc) computes a diagonal scaling matrix  $S$  chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix  $B$  given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of  $B$ ,  $\kappa_2(B)$ , is within a factor  $n$  of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

### 4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **uplo** – Nag\_UploType *Input*

*On entry:* indicates whether the upper or lower triangular part of  $A$  is stored in the array **ab**, as follows:

**uplo** = Nag\_Upper  
The upper triangle of  $A$  is stored.

**uplo** = Nag\_Lower  
The lower triangle of  $A$  is stored.

*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 4: **kd** – Integer *Input*  
*On entry:*  $k_d$ , the number of superdiagonals of the matrix  $A$  if **uplo** = Nag\_Upper, or the number of subdiagonals if **uplo** = Nag\_Lower.  
*Constraint:*  $kd \geq 0$ .
- 5: **ab**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **ab** must be at least  $\max(1, \mathbf{pdab} \times \mathbf{n})$ .  
*On entry:* the upper or lower triangle of the symmetric positive definite band matrix  $A$  whose scaling factors are to be computed.  
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of  $A_{ij}$ , depends on the **order** and **uplo** arguments as follows:
- if **order** = Nag\_ColMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab**[ $k_d + i - j + (j - 1) \times \mathbf{pdab}$ ], for  $j = 1, \dots, n$  and  $i = \max(1, j - k_d), \dots, j$ ;
  - if **order** = Nag\_ColMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab**[ $i - j + (j - 1) \times \mathbf{pdab}$ ], for  $j = 1, \dots, n$  and  $i = j, \dots, \min(n, j + k_d)$ ;
  - if **order** = Nag\_RowMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab**[ $j - i + (i - 1) \times \mathbf{pdab}$ ], for  $i = 1, \dots, n$  and  $j = i, \dots, \min(n, i + k_d)$ ;
  - if **order** = Nag\_RowMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab**[ $k_d + j - i + (i - 1) \times \mathbf{pdab}$ ], for  $i = 1, \dots, n$  and  $j = \max(1, i - k_d), \dots, i$ .
- Only the elements of the array **ab** corresponding to the diagonal elements of  $A$  are referenced. (Row  $(k_d + 1)$  of **ab** when **uplo** = Nag\_Upper, row 1 of **ab** when **uplo** = Nag\_Lower.)
- 6: **pdab** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **ab**.  
*Constraint:*  $\mathbf{pdab} \geq \mathbf{kd} + 1$ .
- 7: **s**[**n**] – double *Output*  
*On exit:* if **fail.code** = NE\_NOERROR, **s** contains the diagonal elements of the scaling matrix  $S$ .
- 8: **scond** – double \* *Output*  
*On exit:* if **fail.code** = NE\_NOERROR, **scond** contains the ratio of the smallest value of **s** to the largest value of **s**. If **scond**  $\geq 0.1$  and **amax** is neither too large nor too small, it is not worth scaling by  $S$ .
- 9: **amax** – double \* *Output*  
*On exit:*  $\max |a_{ij}|$ . If **amax** is very close to overflow or underflow, the matrix  $A$  should be scaled.
- 10: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{kd} = \langle value \rangle$ .

Constraint:  $\mathbf{kd} \geq 0$ .

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

On entry,  $\mathbf{pdab} = \langle value \rangle$ .

Constraint:  $\mathbf{pdab} > 0$ .

### NE\_INT\_2

On entry,  $\mathbf{pdab} = \langle value \rangle$  and  $\mathbf{kd} = \langle value \rangle$ .

Constraint:  $\mathbf{pdab} \geq \mathbf{kd} + 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_MAT\_NOT\_POS\_DEF

The  $\langle value \rangle$ th diagonal element of  $A$  is not positive (and hence  $A$  cannot be positive definite).

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed scale factors will be close to the exact scale factors.

## 8 Parallelism and Performance

nag\_dpbequ (f07hfc) is not threaded in any implementation.

## 9 Further Comments

The complex analogue of this function is nag\_zpbequ (f07htc).

## 10 Example

This example equilibrates the symmetric positive definite matrix  $A$  given by

$$A = \begin{pmatrix} 5.49 & 2.68 \times 10^{10} & 0 & 0 \\ 2.68 \times 10^{10} & 5.63 \times 10^{20} & -2.39 \times 10^{10} & 0 \\ 0 & -2.39 \times 10^{10} & 2.60 & -2.22 \\ 0 & 0 & -2.22 & 5.17 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

### 10.1 Program Text

```
/* nag_dpbequ (f07hfc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double amax, big, scnd, small;
    Integer exit_status = 0, i, j, kd, kd1, kd2, n, pdab;

    /* Arrays */
    double *ab = 0, *s = 0;
    char nag_enum_arg[40];

    /* Nag Types */
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I, J) ab[(J-1)*pdab + kd + I - J]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
    order = Nag_ColMajor;
#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + kd + J - I]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dpbequ (f07hfc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &kd);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &kd);
#endif
}
```

```

#endif
    if (n < 0 || kd < 0) {
        printf("Invalid n or kd\n");
        exit_status = 1;
        goto END;
    }
#ifdef _WIN32
    scanf_s(" %39s%*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n]", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    pdab = kd + 1;
    /* Allocate memory */
    if (!(ab = NAG_ALLOC((kd + 1) * n, double)) || !(s = NAG_ALLOC(n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read the upper or lower triangular part of the band matrix A
     * from data file.
     */
    if (uplo == Nag_Upper) {
        kdl = 0;
        kd2 = kd;
        for (i = 1; i <= n; ++i)
#ifdef _WIN32
            for (j = i; j <= MIN(n, i + kd); ++j)
                scanf_s("%lf", &AB_UPPER(i, j));
#else
            for (j = i; j <= MIN(n, i + kd); ++j)
                scanf("%lf", &AB_UPPER(i, j));
#endif
    }
    else {
        kdl = kd;
        kd2 = 0;
        for (i = 1; i <= n; ++i)
#ifdef _WIN32
            for (j = MAX(1, i - kd); j <= i; ++j)
                scanf_s("%lf", &AB_LOWER(i, j));
#else
            for (j = MAX(1, i - kd); j <= i; ++j)
                scanf("%lf", &AB_LOWER(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    /* Print the matrix A using nag_band_real_mat_print (x04cec). */
    fflush(stdout);
    nag_band_real_mat_print(order, n, n, kdl, kd2, ab, pdab, "Matrix A", 0,
                            &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
              fail.message);
        exit_status = 2;
        goto END;
    }
    printf("\n");

```

```

/* Compute diagonal scaling factors using nag_dpbequ (f07hfc). */
nag_dpbequ(order, uplo, n, kd, ab, pdab, s, &scond, &amax, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dpbequ (f07hfc).\n%s\n", fail.message);
    exit_status = 3;
    goto END;
}
/* Print scond, amax and the scale factors */
printf("scond = %10.1e, amax = %10.1e\n", scond, amax);
printf("\nDiagonal scaling factors\n");
for (i = 0; i < n; ++i)
    printf("%11.1e%s", s[i], i % 7 == 6 ? "\n" : " ");
printf("\n\n");

/* Compute values close to underflow and overflow using
 * nag_real_safe_small_number (x02amc), nag_machine_precision (x02ajc) and
 * nag_real_base (x02bhc)
 */
small = nag_real_safe_small_number / (nag_machine_precision *
                                       nag_real_base);
big = 1.0 / small;
if (scond < 0.1 || amax < small || amax > big) {
    /* Scale A */
    if (uplo == Nag_Upper)
        for (j = 1; j <= n; ++j)
            for (i = MAX(1, j - kd); i <= j; ++i)
                AB_UPPER(i, j) *= s[i - 1] * s[j - 1];
    else
        for (j = 1; j <= n; ++j)
            for (i = j; i <= MIN(n, j + kd); ++i)
                AB_LOWER(i, j) *= s[i - 1] * s[j - 1];

    /* Print the scaled matrix using nag_band_real_mat_print (x04cec). */
    fflush(stdout);
    nag_band_real_mat_print(order, n, n, kd1, kd2, ab, pdab, "Scaled matrix",
                            0, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
              fail.message);
        exit_status = 4;
        goto END;
    }
}
END:
    NAG_FREE(ab);
    NAG_FREE(s);

    return exit_status;
}

#undef AB_UPPER
#undef AB_LOWER

```

## 10.2 Program Data

```

nag_dpbequ (f07hfc) Example Program Data
4 1 : n and kd
Nag_Upper : uplo
5.49 2.68e+10
      5.63e+20 -2.39e+10
              2.60 -2.22
              5.17 : matrix A

```

### 10.3 Program Results

nag\_dpbequ (f07hfc) Example Program Results

Matrix A

	1	2	3	4
1	5.4900e+00	2.6800e+10		
2		5.6300e+20	-2.3900e+10	
3			2.6000e+00	-2.2200e+00
4				5.1700e+00

scond = 6.8e-11, amax = 5.6e+20

Diagonal scaling factors

4.3e-01	4.2e-11	6.2e-01	4.4e-01
---------	---------	---------	---------

Scaled matrix

	1	2	3	4
1	1.0000	0.4821		
2		1.0000	-0.6247	
3			1.0000	-0.6055
4				1.0000

---