

# NAG Library Function Document

## nag\_dpbsv (f07hac)

### 1 Purpose

nag\_dpbsv (f07hac) computes the solution to a real system of linear equations

$$AX = B,$$

where  $A$  is an  $n$  by  $n$  symmetric positive definite band matrix of bandwidth  $(2k_d + 1)$  and  $X$  and  $B$  are  $n$  by  $r$  matrices.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dpbsv (Nag_OrderType order, Nag_UploType uplo, Integer n,
                Integer kd, Integer nrhs, double ab[], Integer pdab, double b[],
                Integer pdb, NagError *fail)
```

### 3 Description

nag\_dpbsv (f07hac) uses the Cholesky decomposition to factor  $A$  as  $A = U^T U$  if **uplo** = Nag\_Upper or  $A = LL^T$  if **uplo** = Nag\_Lower, where  $U$  is an upper triangular band matrix, and  $L$  is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as  $A$ . The factored form of  $A$  is then used to solve the system of equations  $AX = B$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **uplo** – Nag\_UploType *Input*  
*On entry:* if **uplo** = Nag\_Upper, the upper triangle of  $A$  is stored.  
 If **uplo** = Nag\_Lower, the lower triangle of  $A$  is stored.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of linear equations, i.e., the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 4: **kd** – Integer *Input*  
*On entry:*  $k_d$ , the number of superdiagonals of the matrix  $A$  if **uplo** = Nag\_Upper, or the number of subdiagonals if **uplo** = Nag\_Lower.  
*Constraint:*  $kd \geq 0$ .
- 5: **nrhs** – Integer *Input*  
*On entry:*  $r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .  
*Constraint:*  $nrhs \geq 0$ .
- 6: **ab**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **ab** must be at least  $\max(1, \mathbf{pdab} \times \mathbf{n})$ .  
*On entry:* the upper or lower triangle of the symmetric band matrix  $A$ .  
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of  $A_{ij}$ , depends on the **order** and **uplo** arguments as follows:  
if **order** = Nag\_ColMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab**[ $k_d + i - j + (j - 1) \times \mathbf{pdab}$ ], for  $j = 1, \dots, n$  and  $i = \max(1, j - k_d), \dots, j$ ;  
if **order** = Nag\_ColMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab**[ $i - j + (j - 1) \times \mathbf{pdab}$ ], for  $j = 1, \dots, n$  and  $i = j, \dots, \min(n, j + k_d)$ ;  
if **order** = Nag\_RowMajor and **uplo** = Nag\_Upper,  
 $A_{ij}$  is stored in **ab**[ $j - i + (i - 1) \times \mathbf{pdab}$ ], for  $i = 1, \dots, n$  and  $j = i, \dots, \min(n, i + k_d)$ ;  
if **order** = Nag\_RowMajor and **uplo** = Nag\_Lower,  
 $A_{ij}$  is stored in **ab**[ $k_d + j - i + (i - 1) \times \mathbf{pdab}$ ], for  $i = 1, \dots, n$  and  $j = \max(1, i - k_d), \dots, i$ .  
*On exit:* if **fail.code** = NE\_NOERROR, the triangular factor  $U$  or  $L$  from the Cholesky factorization  $A = U^T U$  or  $A = L L^T$  of the band matrix  $A$ , in the same storage format as  $A$ .
- 7: **pdab** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **ab**.  
*Constraint:*  $\mathbf{pdab} \geq \mathbf{kd} + 1$ .
- 8: **b**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **b** must be at least  
 $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.  
The  $(i, j)$ th element of the matrix  $B$  is stored in  
**b**[ $(j - 1) \times \mathbf{pdb} + i - 1$ ] when **order** = Nag\_ColMajor;  
**b**[ $(i - 1) \times \mathbf{pdb} + j - 1$ ] when **order** = Nag\_RowMajor.  
*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .  
*On exit:* if **fail.code** = NE\_NOERROR, the  $n$  by  $r$  solution matrix  $X$ .

- 9:     **pdb** – Integer *Input*  
       On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.  
       Constraints:  
           if **order** = Nag\_ColMajor, **pdb**  $\geq \max(1, \mathbf{n})$ ;  
           if **order** = Nag\_RowMajor, **pdb**  $\geq \max(1, \mathbf{nrhs})$ .
- 10:    **fail** – NagError \* *Input/Output*  
       The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
 See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **kd** =  $\langle value \rangle$ .

Constraint: **kd**  $\geq 0$ .

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **nrhs** =  $\langle value \rangle$ .

Constraint: **nrhs**  $\geq 0$ .

On entry, **pdab** =  $\langle value \rangle$ .

Constraint: **pdab**  $> 0$ .

On entry, **pdb** =  $\langle value \rangle$ .

Constraint: **pdb**  $> 0$ .

### NE\_INT\_2

On entry, **pdab** =  $\langle value \rangle$  and **kd** =  $\langle value \rangle$ .

Constraint: **pdab**  $\geq \mathbf{kd} + 1$ .

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .

Constraint: **pdb**  $\geq \max(1, \mathbf{nrhs})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_MAT\_NOT\_POS\_DEF**

The leading minor of order  $\langle value \rangle$  of  $A$  is not positive definite, so the factorization could not be completed, and the solution has not been computed.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

nag\_dpbsvx (f07hbc) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag\_real\_sym\_posdef\_band\_lin\_solve (f04bfc) solves  $Ax = b$  and returns a forward error bound and condition estimate. nag\_real\_sym\_posdef\_band\_lin\_solve (f04bfc) calls nag\_dpbsv (f07hac) to solve the equations.

**8 Parallelism and Performance**

nag\_dpbsv (f07hac) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dpbsv (f07hac) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

When  $n \gg k$ , the total number of floating-point operations is approximately  $n(k+1)^2 + 4nkr$ , where  $k$  is the number of superdiagonals and  $r$  is the number of right-hand sides.

The complex analogue of this function is nag\_zpbsv (f07hnc).

**10 Example**

This example solves the equations

$$Ax = b,$$

where  $A$  is the symmetric positive definite band matrix

$$A = \begin{pmatrix} 5.49 & 2.68 & 0 & 0 \\ 2.68 & 5.63 & -2.39 & 0 \\ 0 & -2.39 & 2.60 & -2.22 \\ 0 & 0 & -2.22 & 5.17 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 22.09 \\ 9.31 \\ -5.24 \\ 11.83 \end{pmatrix}.$$

Details of the Cholesky factorization of  $A$  are also output.

## 10.1 Program Text

```

/* nag_dpbsv (f07hac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, j, kd, n, nrhs, pdb, pdab;

    /* Arrays */
    double *ab = 0, *b = 0;
    char nag_enum_arg[40];

    /* Nag Types */
    NagError fail;
    Nag_OrderType order;
    Nag_UploType uplo;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I, J) ab[(J-1)*pdab + kd + I - J]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
#define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + kd + J - I]
#define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dpbsv (f07hac) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &kd, &nrhs);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &kd, &nrhs);
#endif
    if (n < 0 || nrhs < 0 || kd < 0) {
        printf("Invalid n, kd or nrhs\n");
    }
}

```

```

        exit_status = 1;
        goto END;
    }
#ifdef _WIN32
    scanf_s("%39s%[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s%[\n]", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    /* Allocate memory */
    if (!(ab = NAG_ALLOC((kd + 1) * n, double)) ||
        !(b = NAG_ALLOC(n * nrhs, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    pdab = kd + 1;
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
#else
    pdb = nrhs;
#endif

    /* Read the upper or lower triangular part of the band matrix A from
     * data file.
     */
    if (uplo == Nag_Upper)
        for (i = 1; i <= n; ++i)
#ifdef _WIN32
            for (j = i; j <= MIN(n, i + kd); ++j)
                scanf_s("%lf", &AB_UPPER(i, j));
#else
            for (j = i; j <= MIN(n, i + kd); ++j)
                scanf("%lf", &AB_UPPER(i, j));
#endif
    else
        for (i = 1; i <= n; ++i)
#ifdef _WIN32
            for (j = MAX(1, i - kd); j <= i; ++j)
                scanf_s("%lf", &AB_LOWER(i, j));
#else
            for (j = MAX(1, i - kd); j <= i; ++j)
                scanf("%lf", &AB_LOWER(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    /* Read b from data file */
    for (i = 1; i <= n; ++i)
#ifdef _WIN32
        for (j = 1; j <= nrhs; ++j)
            scanf_s("%lf", &B(i, j));
#else
        for (j = 1; j <= nrhs; ++j)
            scanf("%lf", &B(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

```

```

/* Solve the equations Ax = b for x using nag_dpbsv (f07hac). */
nag_dpbsv(order, uplo, n, kd, nrhs, ab, pdab, b, pdb, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dpbsv (f07hac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print solution */
printf("Solution\n");
for (i = 1; i <= n; ++i) {
    for (j = 1; j <= nrhs; ++j)
        printf("%11.4f%s", B(i, j), j % 7 == 0 ? "\n" : " ");
    printf("\n");
}

/* Print details of factorization */
printf("\n");
fflush(stdout);
if (uplo == Nag_Upper)
    nag_band_real_mat_print(order, n, n, 0, kd, ab, pdab, "Cholesky factor U",
                            0, &fail);
else
    nag_band_real_mat_print(order, n, n, kd, 0, ab, pdab, "Cholesky factor L",
                            0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
          fail.message);
    exit_status = 1;
}
END:
NAG_FREE(ab);
NAG_FREE(b);

return exit_status;
}

#undef AB_LOWER
#undef AB_UPPER
#undef B

```

## 10.2 Program Data

```

nag_dpbsv (f07hac) Example Program Data
  4          1          1          : n, kd and nrhs
  Nag_Upper          : uplo
  5.49    2.68
        5.63   -2.39
            2.60   -2.22
                5.17 : matrix A
  22.09    9.31   -5.24  11.83 : vector b

```

## 10.3 Program Results

nag\_dpbsv (f07hac) Example Program Results

```

Solution
  5.0000
 -2.0000
 -3.0000
  1.0000

Cholesky factor U
      1          2          3          4
1    2.3431    1.1438
2          2.0789   -1.1497
3          1.1306   -1.9635
4          1.1465

```

---