

# NAG Library Function Document

## nag\_zpocon (f07fuc)

### 1 Purpose

nag\_zpocon (f07fuc) estimates the condition number of a complex Hermitian positive definite matrix  $A$ , where  $A$  has been factorized by nag\_zpotrf (f07frc).

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zpocon (Nag_OrderType order, Nag_UploType uplo, Integer n,
                 const Complex a[], Integer pda, double anorm, double *rcond,
                 NagError *fail)
```

### 3 Description

nag\_zpocon (f07fuc) estimates the condition number (in the 1-norm) of a complex Hermitian positive definite matrix  $A$ :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since  $A$  is Hermitian,  $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$ .

Because  $\kappa_1(A)$  is infinite if  $A$  is singular, the function actually returns an estimate of the **reciprocal** of  $\kappa_1(A)$ .

The function should be preceded by a call to nag\_zhe\_norm (f16ucc) to compute  $\|A\|_1$  and a call to nag\_zpotrf (f07frc) to compute the Cholesky factorization of  $A$ . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate  $\|A^{-1}\|_1$ .

### 4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **uplo** – Nag\_UploType *Input*

*On entry:* specifies how  $A$  has been factorized.

**uplo** = Nag\_Upper

$A = U^H U$ , where  $U$  is upper triangular.

**uplo** = Nag\_Lower

$A = LL^H$ , where  $L$  is lower triangular.

Constraint: **uplo** = Nag\_Upper or Nag\_Lower.

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
 Constraint:  $n \geq 0$ .
- 4: **a**[*dim*] – const Complex *Input*  
**Note:** the dimension, *dim*, of the array **a** must be at least  $\max(1, \mathbf{pda} \times \mathbf{n})$ .  
*On entry:* the Cholesky factor of  $A$ , as returned by nag\_zpotrf (f07frc).
- 5: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.  
 Constraint:  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .
- 6: **anorm** – double *Input*  
*On entry:* the 1-norm of the **original** matrix  $A$ , which may be computed by calling nag\_zhe\_norm (f16ucc) with its argument **norm** = Nag\_OneNorm. **anorm** must be computed either **before** calling nag\_zpotrf (f07frc) or else from a **copy** of the original matrix  $A$ .  
 Constraint:  $\mathbf{anorm} \geq 0.0$ .
- 7: **rcond** – double \* *Output*  
*On exit:* an estimate of the reciprocal of the condition number of  $A$ . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*,  $A$  is singular to working precision.
- 8: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

On entry,  $\mathbf{pda} = \langle value \rangle$ .

Constraint:  $\mathbf{pda} > 0$ .

**NE\_INT\_2**

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq \max(1, \mathbf{n})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_REAL**

On entry, **anorm** =  $\langle value \rangle$ .  
 Constraint: **anorm**  $\geq 0.0$ .

**7 Accuracy**

The computed estimate **rcond** is never less than the true value  $\rho$ , and in practice is nearly always less than  $10\rho$ , although examples can be constructed where **rcond** is much larger.

**8 Parallelism and Performance**

nag\_zpocon (f07fuc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

A call to nag\_zpocon (f07fuc) involves solving a number of systems of linear equations of the form  $Ax = b$ ; the number is usually 5 and never more than 11. Each solution involves approximately  $8n^2$  real floating-point operations but takes considerably longer than a call to nag\_zpotrs (f07fsc) with one right-hand side, because extra care is taken to avoid overflow when  $A$  is approximately singular.

The real analogue of this function is nag\_dpocon (f07fgc).

**10 Example**

This example estimates the condition number in the 1-norm (or  $\infty$ -norm) of the matrix  $A$ , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian positive definite and must first be factorized by nag\_zpotrf (f07frc). The true condition number in the 1-norm is 201.92.

## 10.1 Program Text

```

/* nag_zpocon (f07fuc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double anorm, rcond;
    Integer i, j, n, pda;
    Integer exit_status = 0;
    Nag_UploType uplo;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    char nag_enum_arg[40];
    Complex *a = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zpocon (f07fuc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &n);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &n);
#endif
#ifdef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif

    /* Allocate memory */
    if (!(a = NAG_ALLOC(n * n, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
#ifdef _WIN32

```

```

    scanf_s(" %39s%*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n] ", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    if (uplo == Nag_Upper) {
        for (i = 1; i <= n; ++i) {
            for (j = i; j <= n; ++j)
#ifdef _WIN32
                scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
            else
                scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
        }
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    }
    else {
        for (i = 1; i <= n; ++i) {
            for (j = 1; j <= i; ++j)
#ifdef _WIN32
                scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
            else
                scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
        }
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    }

    /* Compute norm of A */
    /* nag_zhe_norm (f16ucc).
     * 1-norm, infinity-norm, Frobenius norm, largest absolute
     * element, complex Hermitian matrix
     */
    nag_zhe_norm(order, Nag_OneNorm, uplo, n, a, pda, &anorm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zhe_norm (f16ucc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Factorize A */

    /* nag_zpotrf (f07frc).
     * Cholesky factorization of complex Hermitian
     * positive-definite matrix
     */
    nag_zpotrf(order, uplo, n, a, pda, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zpotrf (f07frc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Estimate condition number */
    /* nag_zpocon (f07fuc).
     * Estimate condition number of complex Hermitian
     * positive-definite matrix, matrix already factorized by
     * nag_zpotrf (f07frc)
     */
    nag_zpocon(order, uplo, n, a, pda, anorm, &rcond, &fail);

```

```

if (fail.code != NE_NOERROR) {
    printf("Error from nag_zpocon (f07fuc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
if (rcond >= nag_machine_precision)
    printf("Estimate of condition number =%11.2e\n", 1.0 / rcond);
else
    printf("A is singular to working precision\n");
END:
    NAG_FREE(a);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_zpocon (f07fuc) Example Program Data
4                                     :Value of n
Nag_Lower                           :Value of uplo
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A

```

## 10.3 Program Results

nag\_zpocon (f07fuc) Example Program Results

Estimate of condition number = 1.51e+02

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