

# NAG Library Function Document

## nag\_zgtrfs (f07cvc)

### 1 Purpose

nag\_zgtrfs (f07cvc) computes error bounds and refines the solution to a complex system of linear equations  $AX = B$  or  $A^T X = B$  or  $A^H X = B$ , where  $A$  is an  $n$  by  $n$  tridiagonal matrix and  $X$  and  $B$  are  $n$  by  $r$  matrices, using the  $LU$  factorization returned by nag\_zgttrf (f07cvc) and an initial solution returned by nag\_zgttrs (f07csc). Iterative refinement is used to reduce the backward error as much as possible.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zgtrfs (Nag_OrderType order, Nag_TransType trans, Integer n,
                 Integer nrhs, const Complex dl[], const Complex d[], const Complex du[],
                 const Complex dlf[], const Complex df[], const Complex duf[],
                 const Complex du2[], const Integer ipiv[], const Complex b[],
                 Integer pdb, Complex x[], Integer pdx, double ferr[], double berr[],
                 NagError *fail)
```

### 3 Description

nag\_zgtrfs (f07cvc) should normally be preceded by calls to nag\_zgttrf (f07cvc) and nag\_zgttrs (f07csc). nag\_zgttrf (f07cvc) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix  $A$  as

$$A = PLU,$$

where  $P$  is a permutation matrix,  $L$  is unit lower triangular with at most one nonzero subdiagonal element in each column, and  $U$  is an upper triangular band matrix, with two superdiagonals. nag\_zgttrs (f07csc) then utilizes the factorization to compute a solution,  $\hat{X}$ , to the required equations. Letting  $\hat{x}$  denote a column of  $\hat{X}$ , nag\_zgtrfs (f07cvc) computes a *component-wise backward error*,  $\beta$ , the smallest relative perturbation in each element of  $A$  and  $b$  such that  $\hat{x}$  is the exact solution of a perturbed system

$$(A + E)\hat{x} = b + f, \quad \text{with} \quad |e_{ij}| \leq \beta |a_{ij}|, \quad \text{and} \quad |f_j| \leq \beta |b_j|.$$

The function also estimates a bound for the *component-wise forward error* in the computed solution defined by  $\max |x_i - \hat{x}_i| / \max |\hat{x}_i|$ , where  $x$  is the corresponding column of the exact solution,  $X$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*
- On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.
- Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

- 2:    **trans** – Nag\_TransType *Input*  
*On entry:* specifies the equations to be solved as follows:  
**trans** = Nag\_NoTrans  
           Solve  $AX = B$  for  $X$ .  
**trans** = Nag\_Trans  
           Solve  $A^T X = B$  for  $X$ .  
**trans** = Nag\_ConjTrans  
           Solve  $A^H X = B$  for  $X$ .  
*Constraint:* **trans** = Nag\_NoTrans, Nag\_Trans or Nag\_ConjTrans.
- 3:    **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 4:    **nrhs** – Integer *Input*  
*On entry:*  $r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .  
*Constraint:* **nrhs**  $\geq 0$ .
- 5:    **dl**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **dl** must be at least  $\max(1, n - 1)$ .  
*On entry:* must contain the  $(n - 1)$  subdiagonal elements of the matrix  $A$ .
- 6:    **d**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **d** must be at least  $\max(1, n)$ .  
*On entry:* must contain the  $n$  diagonal elements of the matrix  $A$ .
- 7:    **du**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **du** must be at least  $\max(1, n - 1)$ .  
*On entry:* must contain the  $(n - 1)$  superdiagonal elements of the matrix  $A$ .
- 8:    **dlf**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **dlf** must be at least  $\max(1, n - 1)$ .  
*On entry:* must contain the  $(n - 1)$  multipliers that define the matrix  $L$  of the  $LU$  factorization of  $A$ .
- 9:    **df**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **df** must be at least  $\max(1, n)$ .  
*On entry:* must contain the  $n$  diagonal elements of the upper triangular matrix  $U$  from the  $LU$  factorization of  $A$ .
- 10:   **duf**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **duf** must be at least  $\max(1, n - 1)$ .  
*On entry:* must contain the  $(n - 1)$  elements of the first superdiagonal of  $U$ .
- 11:   **du2**[ $dim$ ] – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **du2** must be at least  $\max(1, n - 2)$ .

*On entry:* must contain the  $(n - 2)$  elements of the second superdiagonal of  $U$ .

12: **ipiv**[*dim*] – const Integer

*Input*

**Note:** the dimension, *dim*, of the array **ipiv** must be at least  $\max(1, \mathbf{n})$ .

*On entry:* must contain the  $n$  pivot indices that define the permutation matrix  $P$ . At the  $i$ th step, row  $i$  of the matrix was interchanged with row **ipiv**[ $i - 1$ ], and **ipiv**[ $i - 1$ ] must always be either  $i$  or  $(i + 1)$ , **ipiv**[ $i - 1$ ] =  $i$  indicating that a row interchange was not performed.

13: **b**[*dim*] – const Complex

*Input*

**Note:** the dimension, *dim*, of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $B$  is stored in

**b**[( $j - 1$ )  $\times$  **pdb** +  $i - 1$ ] when **order** = Nag\_ColMajor;  
**b**[( $i - 1$ )  $\times$  **pdb** +  $j - 1$ ] when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $r$  matrix of right-hand sides  $B$ .

14: **pdb** – Integer

*Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.

*Constraints:*

if **order** = Nag\_ColMajor, **pdb**  $\geq \max(1, \mathbf{n})$ ;  
if **order** = Nag\_RowMajor, **pdb**  $\geq \max(1, \mathbf{nrhs})$ .

15: **x**[*dim*] – Complex

*Input/Output*

**Note:** the dimension, *dim*, of the array **x** must be at least

$\max(1, \mathbf{pdx} \times \mathbf{nrhs})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdx})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $X$  is stored in

**x**[( $j - 1$ )  $\times$  **pdx** +  $i - 1$ ] when **order** = Nag\_ColMajor;  
**x**[( $i - 1$ )  $\times$  **pdx** +  $j - 1$ ] when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $r$  initial solution matrix  $X$ .

*On exit:* the  $n$  by  $r$  refined solution matrix  $X$ .

16: **pdx** – Integer

*Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **x**.

*Constraints:*

if **order** = Nag\_ColMajor, **pdx**  $\geq \max(1, \mathbf{n})$ ;  
if **order** = Nag\_RowMajor, **pdx**  $\geq \max(1, \mathbf{nrhs})$ .

17: **ferr**[**nrhs**] – double

*Output*

*On exit:* estimate of the forward error bound for each computed solution vector, such that  $\|\hat{x}_j - x_j\|_\infty / \|\hat{x}_j\|_\infty \leq \mathbf{ferr}[j - 1]$ , where  $\hat{x}_j$  is the  $j$ th column of the computed solution returned in the array **x** and  $x_j$  is the corresponding column of the exact solution  $X$ . The estimate is almost always a slight overestimate of the true error.

- 18: **berr**[nrhs] – double *Output*  
*On exit:* estimate of the component-wise relative backward error of each computed solution vector  $\hat{x}_j$  (i.e., the smallest relative change in any element of  $A$  or  $B$  that makes  $\hat{x}_j$  an exact solution).
- 19: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
 See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .  
 Constraint: **n**  $\geq 0$ .  
 On entry, **nrhs** =  $\langle value \rangle$ .  
 Constraint: **nrhs**  $\geq 0$ .  
 On entry, **pdb** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $> 0$ .  
 On entry, **pdx** =  $\langle value \rangle$ .  
 Constraint: **pdx**  $> 0$ .

### NE\_INT\_2

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $\geq \max(1, \mathbf{n})$ .  
 On entry, **pdb** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $\geq \max(1, \mathbf{nrhs})$ .  
 On entry, **pdx** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pdx**  $\geq \max(1, \mathbf{n})$ .  
 On entry, **pdx** =  $\langle value \rangle$  and **nrhs** =  $\langle value \rangle$ .  
 Constraint: **pdx**  $\geq \max(1, \mathbf{nrhs})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_{\infty}}{\|x\|_{\infty}} \leq \kappa(A) \frac{\|E\|_{\infty}}{\|A\|_{\infty}},$$

where  $\kappa(A) = \|A^{-1}\|_{\infty}\|A\|_{\infty}$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Function nag\_zgtcon (f07cuc) can be used to estimate the condition number of  $A$ .

## 8 Parallelism and Performance

nag\_zgtrfs (f07cvc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_zgtrfs (f07cvc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations required to solve the equations  $AX = B$  or  $A^T X = B$  or  $A^H X = B$  is proportional to  $nr$ . At most five steps of iterative refinement are performed, but usually only one or two steps are required.

The real analogue of this function is nag\_dgtrfs (f07chc).

## 10 Example

This example solves the equations

$$AX = B,$$

where  $A$  is the tridiagonal matrix

$$A = \begin{pmatrix} -1.3 + 1.3i & 2.0 - 1.0i & 0 & 0 & 0 \\ 1.0 - 2.0i & -1.3 + 1.3i & 2.0 + 1.0i & 0 & 0 \\ 0 & 1.0 + 1.0i & -1.3 + 3.3i & -1.0 + 1.0i & 0 \\ 0 & 0 & 2.0 - 3.0i & -0.3 + 4.3i & 1.0 - 1.0i \\ 0 & 0 & 0 & 1.0 + 1.0i & -3.3 + 1.3i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 2.4 - 5.0i & 2.7 + 6.9i \\ 3.4 + 18.2i & -6.9 - 5.3i \\ -14.7 + 9.7i & -6.0 - 0.6i \\ 31.9 - 7.7i & -3.9 + 9.3i \\ -1.0 + 1.6i & -3.0 + 12.2i \end{pmatrix}.$$

Estimates for the backward errors and forward errors are also output.

## 10.1 Program Text

```

/* nag_zgtrfs (f07cvc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagf16.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, j, n, nrhs, pdb, pdx;

    /* Arrays */
    Complex *b = 0, *d = 0, *df = 0, *dl = 0, *dlf = 0, *du = 0,
             *du2 = 0, *duf = 0, *x = 0;
    double *berr = 0, *ferr = 0;
    Integer *ipiv = 0;

    /* Nag Types */
    NagError fail;
    Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define B(I, J) b[(J - 1) * pdb + I - 1]
    order = Nag_ColMajor;
#else
#define B(I, J) b[(I - 1) * pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zgtrfs (f07cvc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &nrhs);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &nrhs);
#endif
    if (n < 0 || nrhs < 0) {
        printf("Invalid n or nrhs\n");
        exit_status = 1;
        goto END;
    }

    if (!(b = NAG_ALLOC(n * nrhs, Complex)) ||
        !(d = NAG_ALLOC(n, Complex)) ||
        !(df = NAG_ALLOC(n, Complex)) ||
        !(dl = NAG_ALLOC(n - 1, Complex)) ||
        !(dlf = NAG_ALLOC(n - 1, Complex)) ||
        !(du = NAG_ALLOC(n - 1, Complex)) ||
        !(du2 = NAG_ALLOC(n - 2, Complex)) ||
        !(duf = NAG_ALLOC(n - 1, Complex)) ||
        !(x = NAG_ALLOC(n * nrhs, Complex)) ||
        !(berr = NAG_ALLOC(nrhs, double)) ||

```

```

        !(ferr = NAG_ALLOC(nrhs, double)) || !(ipiv = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
    pdx = n;
#else
    pdb = nrhs;
    pdx = nrhs;
#endif

    /* Read the tridiagonal matrix A from data file */
#ifdef _WIN32
    for (i = 0; i < n - 1; ++i)
        scanf_s(" ( %lf , %lf )", &du[i].re, &du[i].im);
#else
    for (i = 0; i < n - 1; ++i)
        scanf(" ( %lf , %lf )", &du[i].re, &du[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    for (i = 0; i < n; ++i)
        scanf_s(" ( %lf , %lf )", &d[i].re, &d[i].im);
#else
    for (i = 0; i < n; ++i)
        scanf(" ( %lf , %lf )", &d[i].re, &d[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    for (i = 0; i < n - 1; ++i)
        scanf_s(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#else
    for (i = 0; i < n - 1; ++i)
        scanf(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    /* Read the right hand matrix B */
    for (i = 1; i <= n; ++i)
        for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
            scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
            scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    /* Copy A into duf, df and dlf */
    for (i = 0; i < n - 1; ++i) {
        duf[i].re = du[i].re, duf[i].im = du[i].im;
        df[i].re = d[i].re, df[i].im = d[i].im;
        dlf[i].re = dl[i].re, dlf[i].im = dl[i].im;
    }

```

```

}
df[n - 1].re = d[n - 1].re, df[n - 1].im = d[n - 1].im;

/* copy B into X using nag_zge_copy (f16tfc). */
nag_zge_copy(order, Nag_NoTrans, n, nrhs, b, pdb, x, pdx, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zge_copy (f16tfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Factorize the copy of the tridiagonal matrix A using
 * nag_zgttrf (f07crc).
 */
nag_zgttrf(n, dlf, df, duf, du2, ipiv, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgttrf (f07crc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Solve the equations AX = B using nag_zgttrs (f07csc). */
nag_zgttrs(order, Nag_NoTrans, n, nrhs, dlf, df, duf, du2, ipiv, x,
           pdx, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgttrs (f07csc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Improve the solution and compute error estimates using
 * nag_zgtrfs (f07cvc).
 */
nag_zgtrfs(order, Nag_NoTrans, n, nrhs, dl, d, du, dlf, df, duf, du2,
           ipiv, b, pdb, x, pdx, ferr, berr, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgtrfs (f07cvc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print the solution using nag_gen_complx_mat_print_comp (x04dbc). */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
                             nrhs, x, pdx, Nag_BracketForm, "%7.4f",
                             "Solution(s)", Nag_IntegerLabels, 0,
                             Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}

/* Print the forward and backward error estimates */
printf("\nBackward errors (machine-dependent)\n");
for (j = 0; j < nrhs; ++j)
    printf("%11.1e%s", berr[j], j % 7 == 6 ? "\n" : " ");

printf("\n\nEstimated forward error bounds (machine-dependent)\n");
for (j = 0; j < nrhs; ++j)
    printf("%11.1e%s", ferr[j], j % 7 == 6 ? "\n" : " ");
printf("\n");
END:
NAG_FREE(b);
NAG_FREE(berr);
NAG_FREE(d);
NAG_FREE(df);
NAG_FREE(dl);
NAG_FREE(dlf);
NAG_FREE(du);
NAG_FREE(du2);

```



```

    NAG_FREE(duf);
    NAG_FREE(ferr);
    NAG_FREE(x);
    NAG_FREE(ipiv);

    return exit_status;
}

#undef B

```

## 10.2 Program Data

```

nag_zgtrfs (f07cvc) Example Program Data
      5          2
      ( 2.0, -1.0) ( 2.0, 1.0) ( -1.0, 1.0) ( 1.0, -1.0) : n, nrhs
      ( -1.3, 1.3) ( -1.3, 1.3) ( -1.3, 3.3) ( -0.3, 4.3) ( -3.3, 1.3) : du
      ( 1.0, -2.0) ( 1.0, 1.0) ( 2.0, -3.0) ( 1.0, 1.0) : d
      ( 2.4, -5.0) ( 2.7, 6.9) : dl
      ( 3.4, 18.2) ( -6.9, -5.3)
      (-14.7, 9.7) ( -6.0, -0.6)
      ( 31.9, -7.7) ( -3.9, 9.3)
      ( -1.0, 1.6) ( -3.0, 12.2) : B

```

## 10.3 Program Results

```

nag_zgtrfs (f07cvc) Example Program Results

Solution(s)
      1          2
1 ( 1.0000, 1.0000) ( 2.0000,-1.0000)
2 ( 3.0000,-1.0000) ( 1.0000, 2.0000)
3 ( 4.0000, 5.0000) (-1.0000, 1.0000)
4 (-1.0000,-2.0000) ( 2.0000, 1.0000)
5 ( 1.0000,-1.0000) ( 2.0000,-2.0000)

Backward errors (machine-dependent)
      3.7e-17      6.7e-17

Estimated forward error bounds (machine-dependent)
      5.4e-14      7.3e-14

```

---