

# NAG Library Function Document

## nag\_zgbtrf (f07brc)

### 1 Purpose

nag\_zgbtrf (f07brc) computes the  $LU$  factorization of a complex  $m$  by  $n$  band matrix.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zgbtrf (Nag_OrderType order, Integer m, Integer n, Integer kl,
                 Integer ku, Complex ab[], Integer pdab, Integer ipiv[], NagError *fail)
```

### 3 Description

nag\_zgbtrf (f07brc) forms the  $LU$  factorization of a complex  $m$  by  $n$  band matrix  $A$  using partial pivoting, with row interchanges. Usually  $m = n$ , and then, if  $A$  has  $k_l$  nonzero subdiagonals and  $k_u$  nonzero superdiagonals, the factorization has the form  $A = PLU$ , where  $P$  is a permutation matrix,  $L$  is a lower triangular matrix with unit diagonal elements and at most  $k_l$  nonzero elements in each column, and  $U$  is an upper triangular band matrix with  $k_l + k_u$  superdiagonals.

Note that  $L$  is not a band matrix, but the nonzero elements of  $L$  can be stored in the same space as the subdiagonal elements of  $A$ .  $U$  is a band matrix but with  $k_l$  additional superdiagonals compared with  $A$ . These additional superdiagonals are created by the row interchanges.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $A$ .  
*Constraint:*  $m \geq 0$ .
- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .

- 4: **kl** – Integer *Input*  
*On entry:*  $k_l$ , the number of subdiagonals within the band of the matrix  $A$ .  
*Constraint:*  $\mathbf{kl} \geq 0$ .
- 5: **ku** – Integer *Input*  
*On entry:*  $k_u$ , the number of superdiagonals within the band of the matrix  $A$ .  
*Constraint:*  $\mathbf{ku} \geq 0$ .
- 6: **ab**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **ab** must be at least  
 $\max(1, \mathbf{pdab} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pdab})$  when **order** = Nag\_RowMajor.  
*On entry:* the  $m$  by  $n$  matrix  $A$ .  
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements  $A_{ij}$ , for row  $i = 1, \dots, m$  and column  $j = \max(1, i - k_l), \dots, \min(n, i + k_u)$ , depends on the **order** argument as follows:  
if **order** = Nag\_ColMajor,  $A_{ij}$  is stored as  $\mathbf{ab}[(j - 1) \times \mathbf{pdab} + \mathbf{kl} + \mathbf{ku} + i - j]$ ;  
if **order** = Nag\_RowMajor,  $A_{ij}$  is stored as  $\mathbf{ab}[(i - 1) \times \mathbf{pdab} + \mathbf{kl} + j - i]$ .  
See Section 9 in nag\_zgbsv (f07bnc) for further details.  
*On exit:* **ab** is overwritten by details of the factorization.  
The elements,  $u_{ij}$ , of the upper triangular band factor  $U$  with  $k_l + k_u$  super-diagonals, and the multipliers,  $l_{ij}$ , used to form the lower triangular factor  $L$  are stored. The elements  $u_{ij}$ , for  $i = 1, \dots, m$  and  $j = i, \dots, \min(n, i + k_l + k_u)$ , and  $l_{ij}$ , for  $i = 1, \dots, m$  and  $j = \max(1, i - k_l), \dots, i$ , are stored where  $A_{ij}$  is stored on entry.
- 7: **pdab** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **ab**.  
*Constraint:*  $\mathbf{pdab} \geq 2 \times \mathbf{kl} + \mathbf{ku} + 1$ .
- 8: **ipiv**[**min(m, n)**] – Integer *Output*  
*On exit:* the pivot indices that define the permutation matrix. At the  $i$ th step, if **ipiv**[ $i - 1$ ]  $> i$  then row  $i$  of the matrix  $A$  was interchanged with row **ipiv**[ $i - 1$ ], for  $i = 1, 2, \dots, \min(m, n)$ . **ipiv**[ $i - 1$ ]  $\leq i$  indicates that, at the  $i$ th step, a row interchange was not required.
- 9: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INT**

On entry, **kl** =  $\langle value \rangle$ .

Constraint: **kl**  $\geq 0$ .

On entry, **ku** =  $\langle value \rangle$ .

Constraint: **ku**  $\geq 0$ .

On entry, **m** =  $\langle value \rangle$ .

Constraint: **m**  $\geq 0$ .

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **pdab** =  $\langle value \rangle$ .

Constraint: **pdab**  $> 0$ .

**NE\_INT\_3**

On entry, **pdab** =  $\langle value \rangle$ , **kl** =  $\langle value \rangle$  and **ku** =  $\langle value \rangle$ .

Constraint: **pdab**  $\geq 2 \times \mathbf{kl} + \mathbf{ku} + 1$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_SINGULAR**

Element  $\langle value \rangle$  of the diagonal is exactly zero. The factorization has been completed, but the factor  $U$  is exactly singular, and division by zero will occur if it is used to solve a system of equations.

**7 Accuracy**

The computed factors  $L$  and  $U$  are the exact factors of a perturbed matrix  $A + E$ , where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$  is a modest linear function of  $k = k_l + k_u + 1$ , and  $\epsilon$  is the *machine precision*. This assumes  $k \ll \min(m, n)$ .

**8 Parallelism and Performance**

nag\_zgbtrf (f07brc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_zgbtrf (f07brc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of real floating-point operations varies between approximately  $8nk_l(k_u + 1)$  and  $8nk_l(k_l + k_u + 1)$ , depending on the interchanges, assuming  $m = n \gg k_l$  and  $n \gg k_u$ .

A call to `nag_zgbtrf (f07brc)` may be followed by calls to the functions:

`nag_zgbtrs (f07bsc)` to solve  $AX = B$ ,  $A^T X = B$  or  $A^H X = B$ ;

`nag_zgbcon (f07buc)` to estimate the condition number of  $A$ .

The real analogue of this function is `nag_dgbtrf (f07bdc)`.

## 10 Example

This example computes the  $LU$  factorization of the matrix  $A$ , where

$$A = \begin{pmatrix} -1.65 + 2.26i & -2.05 - 0.85i & 0.97 - 2.84i & 0.00 + 0.00i \\ 0.00 + 6.30i & -1.48 - 1.75i & -3.99 + 4.01i & 0.59 - 0.48i \\ 0.00 + 0.00i & -0.77 + 2.83i & -1.06 + 1.94i & 3.33 - 1.04i \\ 0.00 + 0.00i & 0.00 + 0.00i & 4.48 - 1.09i & -0.46 - 1.72i \end{pmatrix}.$$

Here  $A$  is treated as a band matrix with one subdiagonal and two superdiagonals.

### 10.1 Program Text

```
/* nag_zgbtrf (f07brc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, ipiv_len, j, kl, ku, m, n, pdab;
    Integer exit_status = 0;
    NagError fail;
    Nag_OrderType order;

    /* Arrays */
    Complex *ab = 0;
    Integer *ipiv = 0;
#ifdef NAG_LOAD_FP
    /* The following line is needed to force the Microsoft linker
       to load floating point support */
    float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */

#ifdef NAG_COLUMN_MAJOR
#define AB(I, J) ab[(J-1)*pdab + kl + ku + I - J]
    order = Nag_ColMajor;
#else
#define AB(I, J) ab[(I-1)*pdab + kl + J - I]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zgbtrf (f07brc) Example Program Results\n\n");
```

```

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m,
            &n, &kl, &ku);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &m,
            &n, &kl, &ku);
#endif
    ipiv_len = MIN(m, n);
    pdab = 2 * kl + ku + 1;

/* Allocate memory */
if (!(ab = NAG_ALLOC((2 * kl + ku + 1) * n, Complex)) ||
    !(ipiv = NAG_ALLOC(ipiv_len, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A from data file */
for (i = 1; i <= m; ++i) {
    for (j = MAX(i - kl, 1); j <= MIN(i + ku, n); ++j)
#ifdef _WIN32
        scanf_s(" ( %lf , %lf )", &AB(i, j).re, &AB(i, j).im);
#else
        scanf(" ( %lf , %lf )", &AB(i, j).re, &AB(i, j).im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Factorize A */
/* nag_zgbtrf (f07brc).
 * LU factorization of complex m by n band matrix
 */
nag_zgbtrf(order, m, n, kl, ku, ab, pdab, ipiv, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgbtrf (f07brc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print details of factorization */
/* nag_band_complx_mat_print_comp (x04dfc).
 * Print complex packed banded matrix (comprehensive)
 */
fflush(stdout);
nag_band_complx_mat_print_comp(order, m, n, kl, kl + ku, ab, pdab,
                               Nag_BracketForm, "%7.4f",
                               "Details of factorization",
                               Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                               80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_band_complx_mat_print_comp (x04dfc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
}
/* Print pivot indices */
printf("\nipiv\n");
for (i = 1; i <= MIN(m, n); ++i)
    printf("%3" NAG_IFMT "%s", ipiv[i - 1],

```

```

        i % 7 == 0 ? "\n" : "
    printf("\n");
END:
    NAG_FREE(ab);
    NAG_FREE(ipiv);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_zgbtrf (f07brc) Example Program Data
  4  4  1  2                                     :Values of M, N, KL and KU
(-1.65, 2.26) (-2.05,-0.85) ( 0.97,-2.84)
( 0.00, 6.30) (-1.48,-1.75) (-3.99, 4.01) ( 0.59,-0.48)
              (-0.77, 2.83) (-1.06, 1.94) ( 3.33,-1.04)
              ( 4.48,-1.09) (-0.46,-1.72) :End of matrix A

```

## 10.3 Program Results

nag\_zgbtrf (f07brc) Example Program Results

```

Details of factorization
              1              2              3              4
1  ( 0.0000, 6.3000) (-1.4800,-1.7500) (-3.9900, 4.0100) ( 0.5900,-0.4800)
2  ( 0.3587, 0.2619) (-0.7700, 2.8300) (-1.0600, 1.9400) ( 3.3300,-1.0400)
3              ( 0.2314, 0.6358) ( 4.9303,-3.0086) (-1.7692,-1.8587)
4              ( 0.7604, 0.2429) ( 0.4338, 0.1233)

ipiv
  2              3              3              4

```

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