

# NAG Library Function Document

## nag\_dgbequ (f07bfc)

### 1 Purpose

nag\_dgbequ (f07bfc) computes diagonal scaling matrices  $D_R$  and  $D_C$  intended to equilibrate a real  $m$  by  $n$  band matrix  $A$  of band width  $(k_l + k_u + 1)$ , and reduce its condition number.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dgbequ (Nag_OrderType order, Integer m, Integer n, Integer kl,
                 Integer ku, const double ab[], Integer pdab, double r[], double c[],
                 double *rowcnd, double *colcnd, double *amax, NagError *fail)
```

### 3 Description

nag\_dgbequ (f07bfc) computes the diagonal scaling matrices. The diagonal scaling matrices are chosen to try to make the elements of largest absolute value in each row and column of the matrix  $B$  given by

$$B = D_R A D_C$$

have absolute value 1. The diagonal elements of  $D_R$  and  $D_C$  are restricted to lie in the safe range  $(\delta, 1/\delta)$ , where  $\delta$  is the value returned by function nag\_real\_safe\_small\_number (X02AMC). Use of these scaling factors is not guaranteed to reduce the condition number of  $A$  but works well in practice.

### 4 References

None.

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $A$ .  
*Constraint:* **m**  $\geq 0$ .
- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $A$ .  
*Constraint:* **n**  $\geq 0$ .
- 4: **kl** – Integer *Input*  
*On entry:*  $k_l$ , the number of subdiagonals of the matrix  $A$ .  
*Constraint:* **kl**  $\geq 0$ .

- 5: **ku** – Integer *Input*  
*On entry:*  $k_u$ , the number of superdiagonals of the matrix  $A$ .  
*Constraint:*  $\mathbf{ku} \geq 0$ .
- 6: **ab**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **ab** must be at least  
 $\max(1, \mathbf{pdab} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pdab})$  when **order** = Nag\_RowMajor.  
*On entry:* the  $m$  by  $n$  band matrix  $A$  whose scaling factors are to be computed.  
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements  $A_{ij}$ , for row  $i = 1, \dots, m$  and column  $j = \max(1, i - k_l), \dots, \min(n, i + k_u)$ , depends on the **order** argument as follows:  
if **order** = Nag\_ColMajor,  $A_{ij}$  is stored as **ab**[( $j - 1$ )  $\times$  **pdab** + **ku** +  $i - j$ ];  
if **order** = Nag\_RowMajor,  $A_{ij}$  is stored as **ab**[( $i - 1$ )  $\times$  **pdab** + **kl** +  $j - i$ ].  
See Section 9 in nag\_dgbv (f07bac) for further details.
- 7: **pdab** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **ab**.  
*Constraint:*  $\mathbf{pdab} \geq \mathbf{kl} + \mathbf{ku} + 1$ .
- 8: **r**[**m**] – double *Output*  
*On exit:* if **fail.code** = NE\_NOERROR or **fail.code** = NE\_MAT\_COL\_ZERO, **r** contains the row scale factors, the diagonal elements of  $D_R$ . The elements of **r** will be positive.
- 9: **c**[**n**] – double *Output*  
*On exit:* if **fail.code** = NE\_NOERROR, **c** contains the column scale factors, the diagonal elements of  $D_C$ . The elements of **c** will be positive.
- 10: **rowcnd** – double \* *Output*  
*On exit:* if **fail.code** = NE\_NOERROR or **fail.code** = NE\_MAT\_COL\_ZERO, **rowcnd** contains the ratio of the smallest value of **r**[ $i - 1$ ] to the largest value of **r**[ $i - 1$ ]. If **rowcnd**  $\geq 0.1$  and **amax** is neither too large nor too small, it is not worth scaling by  $D_R$ .
- 11: **colcnd** – double \* *Output*  
*On exit:* if **fail.code** = NE\_NOERROR, **colcnd** contains the ratio of the smallest value of **c**[ $i - 1$ ] to the largest value of **c**[ $i - 1$ ].  
If **colcnd**  $\geq 0.1$ , it is not worth scaling by  $D_C$ .
- 12: **amax** – double \* *Output*  
*On exit:*  $\max |a_{ij}|$ . If **amax** is very close to overflow or underflow, the matrix  $A$  should be scaled.
- 13: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{kl} = \langle value \rangle$ .

Constraint:  $\mathbf{kl} \geq 0$ .

On entry,  $\mathbf{ku} = \langle value \rangle$ .

Constraint:  $\mathbf{ku} \geq 0$ .

On entry,  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{m} \geq 0$ .

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

On entry,  $\mathbf{pdab} = \langle value \rangle$ .

Constraint:  $\mathbf{pdab} > 0$ .

### NE\_INT\_3

On entry,  $\mathbf{pdab} = \langle value \rangle$ ,  $\mathbf{kl} = \langle value \rangle$  and  $\mathbf{ku} = \langle value \rangle$ .

Constraint:  $\mathbf{pdab} \geq \mathbf{kl} + \mathbf{ku} + 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_MAT\_COL\_ZERO

Column  $\langle value \rangle$  of  $A$  is exactly zero.

### NE\_MAT\_ROW\_ZERO

Row  $\langle value \rangle$  of  $A$  is exactly zero.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed scale factors will be close to the exact scale factors.

## 8 Parallelism and Performance

nag\_dgbequ (f07bfc) is not threaded in any implementation.

## 9 Further Comments

The complex analogue of this function is nag\_zgbequ (f07btc).

## 10 Example

This example equilibrates the band matrix  $A$  given by

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 \times 10^{-10} & 0 \\ -6.98 \times 10^{10} & 2.46 \times 10^{10} & -2.73 & -2.13 \times 10^{10} \\ 0 & 2.56 & 2.46 \times 10^{-10} & 4.07 \\ 0 & 0 & -4.78 \times 10^{-10} & -3.82 \end{pmatrix}.$$

Details of the scaling factors, and the scaled matrix are output.

### 10.1 Program Text

```
/* nag_dgbequ (f07bfc) Example Program.
*
* NAGPRODCODE Version.
*
* Copyright 2016 Numerical Algorithms Group.
*
* Mark 26, 2016.
*/

#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double amax, big, colcnd, rowcnd, small;
    Integer exit_status = 0, i, j, kl, ku, n, pdab;

    /* Arrays */
    double *ab = 0, *c = 0, *r = 0;

    /* Nag Types */
    NagError fail;
    Nag_OrderType order;
    Nag_Boolean scaled = Nag_FALSE;

#ifdef NAG_COLUMN_MAJOR
#define AB(I, J) ab[(J-1)*pdab + ku + I - J]
    order = Nag_ColMajor;
#else
#define AB(I, J) ab[(I-1)*pdab + kl + J - I]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dgbequ (f07bfc) Example Program Results\n\n");
    fflush(stdout);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    #endif
```

```

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kl, &ku);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kl, &ku);
#endif
if (n < 0 || kl < 0 || ku < 0) {
    printf("Invalid n or kl or ku\n");
    exit_status = 1;
    goto END;
}
/* Allocate arrays */
pdab = kl + ku + 1;
if (!(ab = NAG_ALLOC(pdab * n, double)) ||
    !(c = NAG_ALLOC(n, double)) || !(r = NAG_ALLOC(n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read the band matrix A from data file */
for (i = 1; i <= n; ++i)
    for (j = MAX(i - kl, 1); j <= MIN(i + ku, n); ++j)
#ifdef _WIN32
        scanf_s("%lf", &AB(i, j));
#else
        scanf("%lf", &AB(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

/* Print the matrix A using nag_band_real_mat_print (x04cec). */
fflush(stdout);
nag_band_real_mat_print(order, n, n, kl, ku, ab, pdab, "Matrix A", 0,
                        &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
/* Compute row and column scaling factors using nag_dgbequ (f07bfc). */
nag_dgbequ(order, n, n, kl, ku, ab, pdab, r, c, &rowcnd, &colcnd, &amax,
           &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dgbequ (f07bfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print rowcnd, colcnd, amax and the scale factors */
printf("rowcnd = %10.1e, colcnd = %10.1e, amax = %10.1e\n\n", rowcnd,
       colcnd, amax);
printf("Row scale factors\n");
for (i = 1; i <= n; ++i)
    printf("%11.2e%s", r[i - 1], i % 7 == 0 ? "\n" : " ");

printf("\n\nColumn scale factors\n");
for (i = 1; i <= n; ++i)
    printf("%11.2e%s", c[i - 1], i % 7 == 0 ? "\n" : " ");

/* Compute values close to underflow and overflow using
 * nag_real_safe_small_number (x02amc), nag_machine_precision (x02ajc) and
 * nag_real_base (x02bhc)
 */
small = nag_real_safe_small_number / nag_machine_precision * nag_real_base;

```

```

big = 1.0 / small;
if (colcnd < 0.1) {
    scaled = Nag_TRUE;
    /* column scale A */
    for (j = 1; j <= n; ++j)
        for (i = MAX(1, j - ku); i <= MIN(n, j + kl); ++i)
            AB(i, j) *= c[j - 1];
}
if (rowcnd < 0.1 || amax < small || amax > big) {
    /* row scale A */
    scaled = Nag_TRUE;
    for (j = 1; j <= n; ++j)
        for (i = MAX(1, j - ku); i <= MIN(n, j + kl); ++i)
            AB(i, j) *= r[i - 1];
}
if (scaled) {
    /* Print the row and column scaled matrix using
     * nag_band_real_mat_print (x04cec).
     */
    printf("\n\n");
    fflush(stdout);
    nag_band_real_mat_print(order, n, n, kl, ku, ab, pdab, "Scaled matrix",
                           0, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
}
END:
NAG_FREE(ab);
NAG_FREE(c);
NAG_FREE(r);

return exit_status;
}

```

## 10.2 Program Data

nag\_dgbequ (f07bfc) Example Program Data

4	1	2			: n, kl and ku
-2.30e-01	2.54e+00	-3.66e-10			
-6.98e+10	2.46e+10	-2.73e+00	-2.13e+10		
	2.56e+00	2.46e-10	4.07e+00		
		-4.78e-10	-3.82e+00		: matrix A

## 10.3 Program Results

nag\_dgbequ (f07bfc) Example Program Results

Matrix A

	1	2	3	4
1	-2.3000e-01	2.5400e+00	-3.6600e-10	
2	-6.9800e+10	2.4600e+10	-2.7300e+00	-2.1300e+10
3		2.5600e+00	2.4600e-10	4.0700e+00
4			-4.7800e-10	-3.8200e+00

rowcnd = 3.6e-11, colcnd = 1.4e-10, amax = 7.0e+10

Row scale factors

3.94e-01	1.43e-11	2.46e-01	2.62e-01
----------	----------	----------	----------

Column scale factors

1.00e+00	1.00e+00	6.94e+09	1.00e+00
----------	----------	----------	----------

Scaled matrix

	1	2	3	4
1	-0.0906	1.0000	-1.0000	
2	-1.0000	0.3524	-0.2714	-0.3052
3		0.6290	0.4195	1.0000
4			-0.8684	-1.0000

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