

# NAG Library Function Document

## nag\_matop\_real\_tri\_matrix\_sqrt (f01epc)

### 1 Purpose

nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) computes the principal matrix square root,  $A^{1/2}$ , of a real upper quasi-triangular  $n$  by  $n$  matrix  $A$ .

### 2 Specification

```
#include <nag.h>
#include <nagf01.h>

void nag_matop_real_tri_matrix_sqrt (Integer n, double a[], Integer pda,
                                     NagError *fail)
```

### 3 Description

A square root of a matrix  $A$  is a solution  $X$  to the equation  $X^2 = A$ . A nonsingular matrix has multiple square roots. For a matrix with no eigenvalues on the closed negative real line, the principal square root, denoted by  $A^{1/2}$ , is the unique square root whose eigenvalues lie in the open right half-plane.

nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) computes  $A^{1/2}$ , where  $A$  is an upper quasi-triangular matrix, with  $1 \times 1$  and  $2 \times 2$  blocks on the diagonal. Such matrices arise from the Schur factorization of a real general matrix, as computed by nag\_dhseqr (f08pec), for example. nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) does not require  $A$  to be in the canonical Schur form described in nag\_dhseqr (f08pec), it merely requires  $A$  to be upper quasi-triangular.  $A^{1/2}$  then has the same block triangular structure as  $A$ .

The algorithm used by nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) is described in Higham (1987). In addition a blocking scheme described in Deadman *et al.* (2013) is used.

### 4 References

Björck D and Hammarling S (1983) A Schur method for the square root of a matrix *Linear Algebra Appl.* **52/53** 127–140

Deadman E, Higham N J and Ralha R (2013) Blocked Schur Algorithms for Computing the Matrix Square Root *Applied Parallel and Scientific Computing: 11th International Conference, (PARA 2012, Helsinki, Finland)* P. Manninen and P. Úster, Eds *Lecture Notes in Computer Science* **7782** 171–181 Springer–Verlag

Higham N J (1987) Computing real square roots of a real matrix *Linear Algebra Appl.* **88/89** 405–430

Higham N J (2008) *Functions of Matrices: Theory and Computation* SIAM, Philadelphia, PA, USA

### 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 2: **a**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  $\mathbf{pda} \times \mathbf{n}$ .  
The  $(i, j)$ th element of the matrix  $A$  is stored in **a**[( $j - 1$ )  $\times$   $\mathbf{pda}$  +  $i - 1$ ].

*On entry:* the  $n$  by  $n$  upper quasi-triangular matrix  $A$ .

*On exit:* the  $n$  by  $n$  principal matrix square root  $A^{1/2}$ .

3: **pda** – Integer *Input*

*On entry:* the stride separating matrix row elements in the array **a**.

*Constraint:* **pda**  $\geq$  **n**.

4: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_EIGENVALUES

$A$  has negative or vanishing eigenvalues. The principal square root is not defined in this case. nag\_matop\_real\_gen\_matrix\_sqrt (f01enc) or nag\_matop\_complex\_gen\_matrix\_sqrt (f01fnc) may be able to provide further information.

### NE\_INT

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq$  0.

### NE\_INT\_2

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **pda**  $\geq$  **n**.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed square root  $\hat{X}$  satisfies  $\hat{X}^2 = A + \Delta A$ , where  $\|\Delta A\|_F \approx O(\epsilon)n\|\hat{X}\|_F^2$ , where  $\epsilon$  is *machine precision*.

## 8 Parallelism and Performance

`nag_matop_real_tri_matrix_sqrt (f01epc)` is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_matop_real_tri_matrix_sqrt (f01epc)` makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The cost of the algorithm is  $n^3/3$  floating-point operations; see Algorithm 6.7 of Higham (2008).  $O(n)$  of integer allocatable memory is required by the function.

If  $A$  is a full matrix, then `nag_matop_real_gen_matrix_sqrt (f01enc)` should be used to compute the square root. If  $A$  has negative real eigenvalues then `nag_matop_complex_gen_matrix_sqrt (f01fnc)` can be used to return a complex, non-principal square root.

If condition number and residual bound estimates are required, then `nag_matop_real_gen_matrix_cond_sqrt (f01jdc)` should be used. For further discussion of the condition of the matrix square root see Section 6.1 of Higham (2008).

## 10 Example

This example finds the principal matrix square root of the matrix

$$A = \begin{pmatrix} 6 & 4 & -5 & 15 \\ 8 & 6 & -3 & 10 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 4 & 3 \end{pmatrix}.$$

### 10.1 Program Text

```
/* nag_matop_real_tri_matrix_sqrt (f01epc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf01.h>
#include <nagx04.h>

#define A(I,J) a[J*pda + I]

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    Integer i, j, n, pda;
    /* Arrays */
    double *a = 0;
    /* Nag Types */
    Nag_OrderType order = Nag_ColMajor;
    NagError fail;

    INIT_FAIL(fail);

    /* Output preamble */
    printf("nag_matop_real_tri_matrix_sqrt (f01epc) ");
    printf("Example Program Results\n\n");
```

```

    fflush(stdout);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Read in the problem size */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n]", &n);
#else
    scanf("%" NAG_IFMT "%*[\n]", &n);
#endif

    pda = n;
    if (!(a = NAG_ALLOC(pda * n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read in the matrix A from data file */
    for (i = 0; i < n; i++) {
        for (j = 0; j < n; j++) {
#ifdef _WIN32
            scanf_s("%lf", &A(i, j));
#else
            scanf("%lf", &A(i, j));
#endif
        }
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Find matrix square root using
    * nag_matop_real_tri_matrix_sqrt (f01epc)
    * Real upper quasi-triangular matrix square root
    */
    nag_matop_real_tri_matrix_sqrt(n, a, pda, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_matop_real_tri_matrix_sqrt (f01epc)\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }

    /* Print matrix sqrt(A) using
    * nag_gen_real_mat_print (x04cac)
    * Print real general matrix (easy-to-use)
    */
    nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag,
        n, n, a, pda, "sqrt(A)", NULL, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_gen_real_mat_print (x04cac)\n%s\n", fail.message);
        exit_status = 2;
        goto END;
    }

END:
    NAG_FREE(a);
    return exit_status;
}

```

## 10.2 Program Data

nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) Example Program Data

```
4                               :Value of n

6.0      4.0      -5.0      15.0
8.0      6.0      -3.0      10.0
0.0      0.0       3.0      -4.0
0.0      0.0       4.0       3.0  :End of matrix a
```

## 10.3 Program Results

nag\_matop\_real\_tri\_matrix\_sqrt (f01epc) Example Program Results

```
sqrt(A)
      1      2      3      4
1  2.0000e+00  1.0000e+00 -2.0000e+00  3.0000e+00
2  2.0000e+00  2.0000e+00  3.1721e-16  1.0000e+00
3  0.0000e+00  0.0000e+00  2.0000e+00 -1.0000e+00
4  0.0000e+00  0.0000e+00  1.0000e+00  2.0000e+00
```

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