

# NAG Library Function Document

## nag\_1d\_spline\_interpolant (e01bac)

### 1 Purpose

nag\_1d\_spline\_interpolant (e01bac) determines a cubic spline interpolant to a given set of data.

### 2 Specification

```
#include <nag.h>
#include <nage01.h>

void nag_1d_spline_interpolant (Integer m, const double x[],
                                const double y[], Nag_Spline *spline, NagError *fail)
```

### 3 Description

nag\_1d\_spline\_interpolant (e01bac) determines a cubic spline  $s(x)$ , defined in the range  $x_1 \leq x \leq x_m$ , which interpolates (passes exactly through) the set of data points  $(x_i, y_i)$ , for  $i = 1, 2, \dots, m$ , where  $m \geq 4$  and  $x_1 < x_2 < \dots < x_m$ . Unlike some other spline interpolation algorithms, derivative end conditions are not imposed. The spline interpolant chosen has  $m - 4$  interior knots  $\lambda_5, \lambda_6, \dots, \lambda_m$ , which are set to the values of  $x_3, x_4, \dots, x_{m-2}$  respectively. This spline is represented in its B-spline form (see Cox (1975)):

$$s(x) = \sum_{i=1}^m c_i N_i(x)$$

where  $N_i(x)$  denotes the normalized B-spline of degree 3, defined upon the knots  $\lambda_i, \lambda_{i+1}, \dots, \lambda_{i+4}$ , and  $c_i$  denotes its coefficient, whose value is to be determined by the function.

The use of B-splines requires eight additional knots  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_{m+1}, \lambda_{m+2}, \lambda_{m+3}$  and  $\lambda_{m+4}$  to be specified; the function sets the first four of these to  $x_1$  and the last four to  $x_m$ .

The algorithm for determining the coefficients is as described in Cox (1975) except that  $QR$  factorization is used instead of  $LU$  decomposition. The implementation of the algorithm involves setting up appropriate information for the related function nag\_1d\_spline\_fit\_knots (e02bac) followed by a call of that function. (For further details of nag\_1d\_spline\_fit\_knots (e02bac), see the function document.)

Values of the spline interpolant, or of its derivatives or definite integral, can subsequently be computed as detailed in Section 9.

### 4 References

Cox M G (1975) An algorithm for spline interpolation *J. Inst. Math. Appl.* **15** 95–108

Cox M G (1977) A survey of numerical methods for data and function approximation *The State of the Art in Numerical Analysis* (ed D A H Jacobs) 627–668 Academic Press

### 5 Arguments

- 1: **m** – Integer *Input*  
*On entry:*  $m$ , the number of data points.  
*Constraint:*  $m \geq 4$ .

- 2: **x[m]** – const double *Input*  
*On entry:* **x**[*i* – 1] must be set to  $x_i$ , the *i*th data value of the independent variable  $x$ , for  $i = 1, 2, \dots, m$ .  
*Constraint:* **x**[*i*] < **x**[*i* + 1], for  $i = 0, 1, \dots, m - 2$ .
- 3: **y[m]** – const double *Input*  
*On entry:* **y**[*i* – 1] must be set to  $y_i$ , the *i*th data value of the dependent variable  $y$ , for  $i = 1, 2, \dots, m$ .
- 4: **spline** – Nag\_Spline \*  
 Pointer to structure of type Nag\_Spline with the following members:
- n** – Integer *Output*  
*On exit:* the size of the storage internally allocated to **lamda**. This is set to **m** + 4.
- lamda** – double \* *Output*  
*On exit:* the pointer to which storage of size **n** is internally allocated. **lamda**[*i* – 1] contains the *i*th knot, for  $i = 1, 2, \dots, m + 4$ .
- c** – double \* *Output*  
*On exit:* the pointer to which storage of size **n** – 4 is internally allocated. **c**[*i* – 1] contains the coefficient  $c_i$  of the B-spline  $N_i(x)$ , for  $i = 1, 2, \dots, m$ .
- Note that when the information contained in the pointers **lamda** and **c** is no longer of use, or before a new call to nag\_1d\_spline\_interpolant (e01bac) with the same **spline**, you should free this storage using the NAG macro NAG\_FREE. This storage will not have been allocated if this function returns with **fail.code** ≠ NE\_NOERROR.
- 5: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_INT\_ARG\_LT

On entry, **m** =  $\langle value \rangle$ .  
 Constraint: **m** ≥ 4.

### NE\_NOT\_STRICTLY\_INCREASING

The sequence **x** is not strictly increasing: **x**[ $\langle value \rangle$ ] =  $\langle value \rangle$ , **x**[ $\langle value \rangle$ ] =  $\langle value \rangle$ .

## 7 Accuracy

The rounding errors incurred are such that the computed spline is an exact interpolant for a slightly perturbed set of ordinates  $y_i + \delta y_i$ . The ratio of the root-mean-square value of the  $\delta y_i$  to that of the  $y_i$  is no greater than a small multiple of the relative *machine precision*.

## 8 Parallelism and Performance

nag\_1d\_spline\_interpolant (e01bac) is not threaded in any implementation.

## 9 Further Comments

The time taken by `nag_1d_spline_interpolant` (e01bac) is approximately proportional to  $m$ .

All the  $x_i$  are used as knot positions except  $x_2$  and  $x_{m-1}$ . This choice of knots (see Cox (1977)) means that  $s(x)$  is composed of  $m - 3$  cubic arcs as follows. If  $m = 4$ , there is just a single arc space spanning the whole interval  $x_1$  to  $x_4$ . If  $m \geq 5$ , the first and last arcs span the intervals  $x_1$  to  $x_3$  and  $x_{m-2}$  to  $x_m$  respectively. Additionally if  $m \geq 6$ , the  $i$ th arc, for  $i = 2, 3, \dots, m - 4$ , spans the interval  $x_{i+1}$  to  $x_{i+2}$ .

After the call

```
e01bac(m, x, y, &spline, &fail)
```

the following operations may be carried out on the interpolant  $s(x)$ .

The value of  $s(x)$  at  $x = \mathbf{xval}$  can be provided in the variable **sval** by calling the function

```
e02bbc(xval, &sval, &spline, &fail)
```

The values of  $s(x)$  and its first three derivatives at  $x = \mathbf{xval}$  can be provided in the array **sdif** of dimension 4, by the call

```
e02bcc(derivs, xval, sdif, &spline, &fail)
```

Here **derivs** must specify whether the left- or right-hand value of the third derivative is required (see `nag_1d_spline_deriv` (e02bcc) for details). The value of the integral of  $s(x)$  over the range  $x_1$  to  $x_m$  can be provided in the variable **sint** by

```
e02bdc(&spline, &sint, &fail)
```

## 10 Example

The following example program sets up data from 7 values of the exponential function in the interval 0 to 1. `nag_1d_spline_interpolant` (e01bac) is then called to compute a spline interpolant to these data.

The spline is evaluated by `nag_1d_spline_evaluate` (e02bbc), at the data points and at points halfway between each adjacent pair of data points, and the spline values and the values of  $e^x$  are printed out.

### 10.1 Program Text

```
/* nag_1d_spline_interpolant (e01bac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <math.h>
#include <nag_stdlib.h>
#include <nage01.h>
#include <nage02.h>

#define MMAX 7

int main(void)
{
    Integer exit_status = 0, i, j, m = MMAX;
    NagError fail;
    Nag_Spline spline;
    double fit, *x = 0, xarg, *y = 0;

    INIT_FAIL(fail);

    /* Initialize spline */
    spline.lamda = 0;
```

```

spline.c = 0;

printf("nag_ld_spline_interpolant (e01bac) Example Program Results\n");

if (m >= 1) {
    if (!(y = NAG_ALLOC(m, double)) || !(x = NAG_ALLOC(m, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
}
else {
    exit_status = 1;
    return exit_status;
}

x[0] = 0.0;
x[1] = 0.2;
x[2] = 0.4;
x[3] = 0.6;
x[4] = 0.75;
x[5] = 0.9;
x[6] = 1.0;

for (i = 0; i < m; ++i)
    y[i] = exp(x[i]);
/* nag_ld_spline_interpolant (e01bac).
 * Interpolating function, cubic spline interpolant, one
 * variable
 */
nag_ld_spline_interpolant(m, x, y, &spline, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ld_spline_interpolant (e01bac).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("\nNumber of distinct knots = %" NAG_IFMT "\n\n", m - 2);
printf("Distinct knots located at \n\n");
for (j = 3; j < m + 1; j++)
    printf("%8.4f%s", spline.lamda[j], (j - 3) % 5 == 4
        || j == m ? "\n" : " ");
printf("\n\n      J      B-spline coeff c\n\n");
for (j = 0; j < m; ++j)
    printf("      %" NAG_IFMT " %13.4f\n", j + 1, spline.c[j]);
printf("\n      J      Abscissa      Ordinate      Spline\n\n");
for (j = 0; j < m; ++j) {
    /* nag_ld_spline_evaluate (e02bbc).
     * Evaluation of fitted cubic spline, function only
     */
    nag_ld_spline_evaluate(x[j], &fit, &spline, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ld_spline_evaluate (e02bbc).\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }
}

printf("      %" NAG_IFMT " %13.4f      %13.4f      %13.4f\n",
    j + 1, x[j], y[j], fit);
if (j < m - 1) {
    xarg = (x[j] + x[j + 1]) * 0.5;
    /* nag_ld_spline_evaluate (e02bbc), see above. */
    nag_ld_spline_evaluate(xarg, &fit, &spline, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ld_spline_evaluate (e02bbc).\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }
}

```

```

    }
    printf("          %13.4f          %13.4f\n", xarg, fit);
  }
}
/* Free memory allocated by nag_ld_spline_interpolant (e01bac) */
END:
  NAG_FREE(y);
  NAG_FREE(x);
  NAG_FREE(spline.lamda);
  NAG_FREE(spline.c);
  return exit_status;
}

```

## 10.2 Program Data

None.

## 10.3 Program Results

nag\_ld\_spline\_interpolant (e01bac) Example Program Results

Number of distinct knots = 5

Distinct knots located at

0.0000    0.4000    0.6000    0.7500    1.0000

J      B-spline coeff c

1	1.0000
2	1.1336
3	1.3726
4	1.7827
5	2.1744
6	2.4918
7	2.7183

J	Abscissa	Ordinate	Spline
1	0.0000	1.0000	1.0000
	0.1000		1.1052
2	0.2000	1.2214	1.2214
	0.3000		1.3498
3	0.4000	1.4918	1.4918
	0.5000		1.6487
4	0.6000	1.8221	1.8221
	0.6750		1.9640
5	0.7500	2.1170	2.1170
	0.8250		2.2819
6	0.9000	2.4596	2.4596
	0.9500		2.5857
7	1.0000	2.7183	2.7183

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