

# NAG Library Function Document

## nag\_pde\_bs\_1d (d03ncc)

### 1 Purpose

nag\_pde\_bs\_1d (d03ncc) solves the Black–Scholes equation for financial option pricing using a finite difference scheme.

### 2 Specification

```
#include <nag.h>
#include <nagd03.h>

void nag_pde_bs_1d (Nag_OptionType kopt, double x, Nag_MeshType mesh,
    Integer ns, double s[], Integer nt, double t[],
    const Nag_Boolean tdparr[], const double r[], const double q[],
    const double sigma[], double alpha, Integer ntkeep, double f[],
    double theta[], double delta[], double gamma[], double lambda[],
    double rho[], NagError *fail)
```

### 3 Description

nag\_pde\_bs\_1d (d03ncc) solves the Black–Scholes equation (see Hull (1989) and Wilmott *et al.* (1995))

$$\frac{\partial f}{\partial t} + (r - q)S \frac{\partial f}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} = rf \quad (1)$$

$$S_{\min} < S < S_{\max}, \quad t_{\min} < t < t_{\max}, \quad (2)$$

for the value  $f$  of a European or American, put or call stock option, with exercise price  $X$ . In equation (1)  $t$  is time,  $S$  is the stock price,  $r$  is the risk free interest rate,  $q$  is the continuous dividend, and  $\sigma$  is the stock volatility. According to the values in the array **tdpar**, the arguments  $r$ ,  $q$  and  $\sigma$  may each be either constant or functions of time. The function also returns values of various Greeks.

nag\_pde\_bs\_1d (d03ncc) uses a finite difference method with a choice of time-stepping schemes. The method is explicit for **alpha** = 0.0 and implicit for nonzero values of **alpha**. Second order time accuracy can be obtained by setting **alpha** = 0.5. According to the value of the argument **mesh** the finite difference mesh may be either uniform, or user-defined in both  $S$  and  $t$  directions.

### 4 References

Hull J (1989) *Options, Futures and Other Derivative Securities* Prentice–Hall

Wilmott P, Howison S and Dewynne J (1995) *The Mathematics of Financial Derivatives* Cambridge University Press

### 5 Arguments

1: **kopt** – Nag\_OptionType Input

*On entry:* specifies the kind of option to be valued.

**kopt** = Nag\_EuropeanCall  
A European call option.

**kopt** = Nag\_AmericanCall  
An American call option.

**kopt** = Nag\_EuropeanPut  
A European put option.

**kopt** = Nag\_AmericanPut  
An American put option.

*Constraint:* **kopt** = Nag\_EuropeanCall, Nag\_AmericanCall, Nag\_EuropeanPut or Nag\_AmericanPut.

2: **x** – double *Input*  
*On entry:* the exercise price  $X$ .

3: **mesh** – Nag\_MeshType *Input*  
*On entry:* indicates the type of finite difference mesh to be used:  
**mesh** = Nag\_UniformMesh  
Uniform mesh.  
**mesh** = Nag\_CustomMesh  
Custom mesh supplied by you.  
*Constraint:* **mesh** = Nag\_UniformMesh or Nag\_CustomMesh.

4: **ns** – Integer *Input*  
*On entry:* the number of stock prices to be used in the finite difference mesh.  
*Constraint:* **ns**  $\geq 2$ .

5: **s[ns]** – double *Input/Output*  
*On entry:* if **mesh** = Nag\_CustomMesh, **s**[ $i - 1$ ] must contain the  $i$ th stock price in the mesh, for  $i = 1, 2, \dots, \mathbf{ns}$ . These values should be in increasing order, with **s**[0] =  $S_{\min}$  and **s**[**ns** - 1] =  $S_{\max}$ .  
If **mesh** = Nag\_UniformMesh, **s**[0] must be set to  $S_{\min}$  and **s**[**ns** - 1] to  $S_{\max}$ , but **s**[1], **s**[2], ..., **s**[**ns** - 2] need not be initialized, as they will be set internally by the function in order to define a uniform mesh.  
*On exit:* if **mesh** = Nag\_UniformMesh, the elements of **s** define a uniform mesh over  $[S_{\min}, S_{\max}]$ .  
If **mesh** = Nag\_CustomMesh, the elements of **s** are unchanged.  
*Constraints:*

if **mesh** = Nag\_CustomMesh, **s**[0]  $\geq 0.0$  and **s**[ $i - 1$ ] < **s**[ $i$ ], for  $i = 1, 2, \dots, \mathbf{ns} - 1$ ;  
if **mesh** = Nag\_UniformMesh,  $0.0 \leq \mathbf{s}[0] < \mathbf{s}[\mathbf{ns} - 1]$ .

6: **nt** – Integer *Input*  
*On entry:* the number of time-steps to be used in the finite difference method.  
*Constraint:* **nt**  $\geq 2$ .

7: **t[nt]** – double *Input/Output*  
*On entry:* if **mesh** = Nag\_CustomMesh then **t**[ $j - 1$ ] must contain the  $j$ th time in the mesh, for  $j = 1, 2, \dots, \mathbf{nt}$ . These values should be in increasing order, with **t**[0] =  $t_{\min}$  and **t**[**nt** - 1] =  $t_{\max}$ .  
If **mesh** = Nag\_UniformMesh then **t**[0] must be set to  $t_{\min}$  and **t**[**nt** - 1] to  $t_{\max}$ , but **t**[1], **t**[2], ..., **t**[**nt** - 2] need not be initialized, as they will be set internally by the function in order to define a uniform mesh.  
*On exit:* if **mesh** = Nag\_UniformMesh, the elements of **t** define a uniform mesh over  $[t_{\min}, t_{\max}]$ .  
If **mesh** = Nag\_CustomMesh, the elements of **t** are unchanged.

Constraints:

if **mesh** = Nag\_CustomMesh,  $\mathbf{t}[0] \geq 0.0$  and  $\mathbf{t}[j-1] < \mathbf{t}[j]$ , for  $j = 1, 2, \dots, \mathbf{nt} - 1$ ;  
 if **mesh** = Nag\_UniformMesh,  $0.0 \leq \mathbf{t}[0] < \mathbf{t}[\mathbf{nt} - 1]$ .

8: **tdpar**[3] – const Nag\_Boolean Input

*On entry:* specifies whether or not various arguments are time-dependent. More precisely,  $r$  is time-dependent if **tdpar**[0] = Nag\_TRUE and constant otherwise. Similarly, **tdpar**[1] specifies whether  $q$  is time-dependent and **tdpar**[2] specifies whether  $\sigma$  is time-dependent.

9: **r**[*dim*] – const double Input

**Note:** the dimension, *dim*, of the array **r** must be at least

**nt** when **tdpar**[0] = Nag\_TRUE;  
 1 otherwise.

*On entry:* if **tdpar**[0] = Nag\_TRUE then **r**[ $j-1$ ] must contain the value of the risk-free interest rate  $r(t)$  at the  $j$ th time in the mesh, for  $j = 1, 2, \dots, \mathbf{nt}$ .

If **tdpar**[0] = Nag\_FALSE then **r**[0] must contain the constant value of the risk-free interest rate  $r$ . The remaining elements need not be set.

10: **q**[*dim*] – const double Input

**Note:** the dimension, *dim*, of the array **q** must be at least

**nt** when **tdpar**[1] = Nag\_TRUE;  
 1 otherwise.

*On entry:* if **tdpar**[1] = Nag\_TRUE then **q**[ $j-1$ ] must contain the value of the continuous dividend  $q(t)$  at the  $j$ th time in the mesh, for  $j = 1, 2, \dots, \mathbf{nt}$ .

If **tdpar**[1] = Nag\_FALSE then **q**[0] must contain the constant value of the continuous dividend  $q$ . The remaining elements need not be set.

11: **sigma**[*dim*] – const double Input

**Note:** the dimension, *dim*, of the array **sigma** must be at least

**nt** when **tdpar**[2] = Nag\_TRUE;  
 1 otherwise.

*On entry:* if **tdpar**[2] = Nag\_TRUE then **sigma**[ $j-1$ ] must contain the value of the volatility  $\sigma(t)$  at the  $j$ th time in the mesh, for  $j = 1, 2, \dots, \mathbf{nt}$ .

If **tdpar**[2] = Nag\_FALSE then **sigma**[0] must contain the constant value of the volatility  $\sigma$ . The remaining elements need not be set.

12: **alpha** – double Input

*On entry:* the value of  $\lambda$  to be used in the time-stepping scheme. Typical values include:

**alpha** = 0.0  
 Explicit forward Euler scheme.

**alpha** = 0.5  
 Implicit Crank–Nicolson scheme.

**alpha** = 1.0  
 Implicit backward Euler scheme.

The value 0.5 gives second-order accuracy in time. Values greater than 0.5 give unconditional stability. Since 0.5 is at the limit of unconditional stability this value does not damp oscillations.

*Suggested value:* **alpha** = 0.55.

*Constraint:*  $0.0 \leq \mathbf{alpha} \leq 1.0$ .

13: **ntkeep** – Integer *Input*

*On entry:* the number of solutions to be stored in the time direction. The function calculates the solution backwards from  $\mathbf{t}[\mathbf{nt} - 1]$  to  $\mathbf{t}[0]$  at all times in the mesh. These time solutions and the corresponding Greeks will be stored at times  $\mathbf{t}[i - 1]$ , for  $i = 1, 2, \dots, \mathbf{ntkeep}$ , in the arrays **f**, **theta**, **delta**, **gamma**, **lambda** and **rho**. Other time solutions will be discarded. To store all time solutions set **ntkeep** = **nt**.

*Constraint:*  $1 \leq \mathbf{ntkeep} \leq \mathbf{nt}$ .

14: **f**[**ns** × **ntkeep**] – double *Output*

*On exit:* **f**[**ns** × ( $j - 1$ ) +  $i - 1$ ], for  $i = 1, 2, \dots, \mathbf{ns}$  and  $j = 1, 2, \dots, \mathbf{ntkeep}$ , contains the value  $f$  of the option at the  $i$ th mesh point  $\mathbf{s}[i - 1]$  at time  $\mathbf{t}[j - 1]$ .

15: **theta**[**ns** × **ntkeep**] – double *Output*

16: **delta**[**ns** × **ntkeep**] – double *Output*

17: **gamma**[**ns** × **ntkeep**] – double *Output*

18: **lambda**[**ns** × **ntkeep**] – double *Output*

19: **rho**[**ns** × **ntkeep**] – double *Output*

*On exit:* the values of various Greeks at the  $i$ th mesh point  $\mathbf{s}[i - 1]$  at time  $\mathbf{t}[j - 1]$ , as follows:

$$\mathbf{theta}[\mathbf{ns} \times (j - 1) + i - 1] = \frac{\partial f}{\partial t}, \quad \mathbf{delta}[\mathbf{ns} \times (j - 1) + i - 1] = \frac{\partial f}{\partial S}, \quad \mathbf{gamma}[\mathbf{ns} \times (j - 1) + i - 1] = \frac{\partial^2 f}{\partial S^2},$$

$$\mathbf{lambda}[\mathbf{ns} \times (j - 1) + i - 1] = \frac{\partial f}{\partial \sigma}, \quad \mathbf{rho}[\mathbf{ns} \times (j - 1) + i - 1] = \frac{\partial f}{\partial r}.$$

20: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **ns** =  $\langle value \rangle$ .

Constraint: **ns** ≥ 2.

On entry, **nt** =  $\langle value \rangle$ .

Constraint: **nt** ≥ 2.

On entry, **ntkeep** =  $\langle value \rangle$ .

Constraint: **ntkeep** ≥ 1.

### NE\_INT\_2

On entry, **ntkeep** =  $\langle value \rangle$  and **nt** =  $\langle value \rangle$ .

Constraint: **ntkeep** ≤ **nt**.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_STRICTLY\_INCREASING**

On entry,  $\mathbf{s}[I] \leq \mathbf{s}[I - 1]$  in custom mesh:  $I = \langle \text{value} \rangle$ .

On entry,  $\mathbf{t}[J] \leq \mathbf{t}[J - 1]$  in custom mesh:  $J = \langle \text{value} \rangle$ .

**NE\_REAL**

On entry,  $\mathbf{alpha} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{alpha} \leq 1.0$ .

On entry,  $\mathbf{alpha} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{alpha} \geq 0.0$ .

On entry,  $\mathbf{s}[0] < 0.0$ :  $\mathbf{s}[0] = \langle \text{value} \rangle$ .

On entry,  $\mathbf{t}[0] < 0.0$ :  $\mathbf{t}[0] = \langle \text{value} \rangle$ .

**NE\_REAL\_2**

On entry,  $\mathbf{s}[\mathbf{ns} - 1] = \langle \text{value} \rangle$  and  $\mathbf{s}[0] = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{s}[\mathbf{ns} - 1] > \mathbf{s}[0]$ .

On entry,  $\mathbf{t}[\mathbf{nt} - 1] = \langle \text{value} \rangle$  and  $\mathbf{t}[0] = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{t}[\mathbf{nt} - 1] > \mathbf{t}[0]$ .

**7 Accuracy**

The accuracy of the solution  $f$  and the various derivatives returned by the function is dependent on the values of  $\mathbf{ns}$  and  $\mathbf{nt}$  supplied, the distribution of the mesh points, and the value of  $\mathbf{alpha}$  chosen. For most choices of  $\mathbf{alpha}$  the solution has a truncation error which is second-order accurate in  $S$  and first order accurate in  $t$ . For  $\mathbf{alpha} = 0.5$  the truncation error is also second-order accurate in  $t$ .

The simplest approach to improving the accuracy is to increase the values of both  $\mathbf{ns}$  and  $\mathbf{nt}$ .

**8 Parallelism and Performance**

nag\_pde\_bs\_1d (d03ncc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_pde\_bs\_1d (d03ncc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

### 9.1 Timing

Each time-step requires the construction and solution of a tridiagonal system of linear equations. To calculate each of the derivatives **lambda** and **rho** requires a repetition of the entire solution process. The time taken for a call to the function is therefore proportional to  $ns \times nt$ .

### 9.2 Algorithmic Details

nag\_pde\_bs\_1d (d03ncc) solves equation (1) using a finite difference method. The solution is computed backwards in time from  $t_{\max}$  to  $t_{\min}$  using a  $\lambda$  scheme, which is implicit for all nonzero values of  $\lambda$ , and is unconditionally stable for values of  $\lambda > 0.5$ . For each time-step a tridiagonal system is constructed and solved to obtain the solution at the earlier time. For the explicit scheme ( $\lambda = 0$ ) this tridiagonal system degenerates to a diagonal matrix and is solved trivially. For American options the solution at each time-step is inspected to check whether early exercise is beneficial, and amended accordingly.

To compute the arrays **lambda** and **rho**, which are derivatives of the stock value  $f$  with respect to the problem arguments  $\sigma$  and  $r$  respectively, the entire solution process is repeated with perturbed values of these arguments.

## 10 Example

This example, taken from Hull (1989), solves the one-dimensional Black–Scholes equation for valuation of a 5-month American put option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate is 10% per annum, and the stock volatility is 40% per annum.

A fully implicit backward Euler scheme is used, with a mesh of 20 stock price intervals and 10 time intervals.

### 10.1 Program Text

```
/* nag_pde_bs_1d (d03ncc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <string.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd03.h>

#define F(I, J)      f[ns*((J) -1)+(I) -1]
#define THETA(I, J)  theta[ns*((J) -1)+(I) -1]
#define DELTA(I, J)  delta[ns*((J) -1)+(I) -1]
#define GAMMA(I, J)  gamma[ns*((J) -1)+(I) -1]
#define LAMBDA(I, J) lambda[ns*((J) -1)+(I) -1]
#define RHO(I, J)    rho[ns*((J) -1)+(I) -1]

int main(void)
{
    double alpha, x;
    Integer i, igreek, j, ns, nt, ntkeep, exit_status;
    double *delta = 0, *f = 0, *gamma = 0, *lambda = 0, q[3], r[3],
           *rho = 0, *s = 0;
    double sigma[3], *t = 0, *theta = 0, smin, smax, tmin, tmax;
    Nag_Boolean gprnt[5] = { Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE };
    Nag_Boolean tdp[3];
    const char *gname[5] = { "Theta", "Delta", "Gamma", "Lambda", "Rho" };
    NagError fail;
```

```

INIT_FAIL(fail);

printf("nag_pde_bs_1d (d03ncc) Example Program Results\n\n");

/* Skip heading in data file */

#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
exit_status = 0;

/* Read problem parameters */

#ifdef _WIN32
    scanf_s("%lf", &x);
#else
    scanf("%lf", &x);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "", &ns, &nt);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "", &ns, &nt);
#endif
#ifdef _WIN32
    scanf_s("%lf%lf", &smin, &smax);
#else
    scanf("%lf%lf", &smin, &smax);
#endif
#ifdef _WIN32
    scanf_s("%lf%lf", &tmin, &tmax);
#else
    scanf("%lf%lf", &tmin, &tmax);
#endif
#ifdef _WIN32
    scanf_s("%lf", &alpha);
#else
    scanf("%lf", &alpha);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &ntkeep);
#else
    scanf("%" NAG_IFMT "", &ntkeep);
#endif

/* Allocate memory */

if (!(s = NAG_ALLOC(ns, double)) ||
    !(t = NAG_ALLOC(nt, double)) ||
    !(f = NAG_ALLOC(ns * ntkeep, double)) ||
    !(theta = NAG_ALLOC(ns * ntkeep, double)) ||
    !(delta = NAG_ALLOC(ns * ntkeep, double)) ||
    !(gamma = NAG_ALLOC(ns * ntkeep, double)) ||
    !(lambda = NAG_ALLOC(ns * ntkeep, double)) ||
    !(rho = NAG_ALLOC(ns * ntkeep, double)))
{
    printf("Allocation failure\n");
    exit_status = 1;
    goto END;
}

/* Set up input parameters for nag_pde_bs_1d (d03ncc) */

s[0] = smin;
s[ns - 1] = smax;
t[0] = tmin;
t[nt - 1] = tmax;
tdpar[0] = Nag_FALSE;
tdpar[1] = Nag_FALSE;

```

```

tdpar[2] = Nag_FALSE;
q[0] = 0.0;
r[0] = 0.10;
sigma[0] = 0.4;

/* Call Black-Scholes solver */

/* nag_pde_bs_1d (d03ncc).
 * Finite difference solution of the Black-Scholes equations
 */
nag_pde_bs_1d(Nag_AmericanPut, x, Nag_UniformMesh, ns, s,
              nt, t, tdpar, r, q, sigma, alpha, ntkeep, f,
              theta, delta, gamma, lambda, rho, &fail);

if (fail.code != NE_NOERROR) {
    printf("Error from nag_pde_bs_1d (d03ncc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Output option values */

printf("\nOption Values\n");
printf("-----\n");
printf("%14s | %s\n", "Stock Price", "Time to Maturity (months)");
printf("%14s | ", "");
for (i = 0; i < ntkeep; i++)
    printf(" %13.4e", 12.0 * (t[nt - 1] - t[i]));
printf("\n");
for (i = 0; i < 74; i++)
    printf("-");
printf("\n");
for (i = 1; i <= ns; i++) {
    printf(" %13.4e | ", s[i - 1]);
    for (j = 1; j <= ntkeep; j++)
        printf(" %13.4e", F(i, j));
    printf("\n");
}

for (igreek = 0; igreek < 5; igreek++) {
    if (!gprnt[igreek])
        continue;

    printf("\n%s\n", gname[igreek]);
    for (i = 0; i < (Integer) strlen(gname[igreek]); i++)
        printf("-");
    printf("\n%14s | %s\n", "Stock Price", "Time to Maturity (months)");
    printf("%14s | ", "");
    for (i = 0; i < ntkeep; i++)
        printf(" %13.4e", 12.0 * (t[nt - 1] - t[i]));
    printf("\n");
    for (i = 0; i < 74; i++)
        printf("-");
    printf("\n");

    for (i = 1; i <= ns; i++) {
        printf(" %13.4e | ", s[i - 1]);
        switch (igreek) {
            case 0:
                for (j = 1; j <= ntkeep; j++)
                    printf(" %13.4e", THETA(i, j));
                break;
            case 1:
                for (j = 1; j <= ntkeep; j++)
                    printf(" %13.4e", DELTA(i, j));
                break;
            case 2:
                for (j = 1; j <= ntkeep; j++)
                    printf(" %13.4e", GAMMA(i, j));
                break;
            case 3:

```



```

        for (j = 1; j <= ntkeep; j++)
            printf(" %13.4e", LAMBDA(i, j));
        break;
    case 4:
        for (j = 1; j <= ntkeep; j++)
            printf(" %13.4e", RHO(i, j));
        break;
    default:
        break;
    }
    printf("\n");
}
}

END:
    NAG_FREE(s);
    NAG_FREE(t);
    NAG_FREE(f);
    NAG_FREE(theta);
    NAG_FREE(delta);
    NAG_FREE(gamma);
    NAG_FREE(lambda);
    NAG_FREE(rho);

    return exit_status;
}

```

## 10.2 Program Data

nag\_pde\_bs\_1d (d03ncc) Example Program Data

```

50.
21  11
0.0  100.
0.0  0.4166667
1.0
4

```

## 10.3 Program Results

nag\_pde\_bs\_1d (d03ncc) Example Program Results

Option Values

Stock Price	Time to Maturity (months)			
	5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00
0.0000e+00	5.0000e+01	5.0000e+01	5.0000e+01	5.0000e+01
5.0000e+00	4.5000e+01	4.5000e+01	4.5000e+01	4.5000e+01
1.0000e+01	4.0000e+01	4.0000e+01	4.0000e+01	4.0000e+01
1.5000e+01	3.5000e+01	3.5000e+01	3.5000e+01	3.5000e+01
2.0000e+01	3.0000e+01	3.0000e+01	3.0000e+01	3.0000e+01
2.5000e+01	2.5000e+01	2.5000e+01	2.5000e+01	2.5000e+01
3.0000e+01	2.0000e+01	2.0000e+01	2.0000e+01	2.0000e+01
3.5000e+01	1.5000e+01	1.5000e+01	1.5000e+01	1.5000e+01
4.0000e+01	1.0154e+01	1.0096e+01	1.0046e+01	1.0012e+01
4.5000e+01	6.5848e+00	6.4424e+00	6.2916e+00	6.1306e+00
5.0000e+01	4.0672e+00	3.8785e+00	3.6729e+00	3.4463e+00
5.5000e+01	2.4264e+00	2.2423e+00	2.0454e+00	1.8336e+00
6.0000e+01	1.4174e+00	1.2662e+00	1.1096e+00	9.4813e-01
6.5000e+01	8.1951e-01	7.0724e-01	5.9532e-01	4.8515e-01
7.0000e+01	4.7241e-01	3.9411e-01	3.1904e-01	2.4845e-01
7.5000e+01	2.7257e-01	2.2016e-01	1.7174e-01	1.2815e-01
8.0000e+01	1.5725e-01	1.2328e-01	9.2935e-02	6.6682e-02
8.5000e+01	8.9662e-02	6.8478e-02	5.0100e-02	3.4731e-02
9.0000e+01	4.8449e-02	3.6251e-02	2.5901e-02	1.7469e-02
9.5000e+01	2.1100e-02	1.5584e-02	1.0968e-02	7.2680e-03
1.0000e+02	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00

Theta

-----					
Stock Price	Time to Maturity (months)				
	5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00	
-----					
0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
5.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.5000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.5000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.5000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
4.0000e+01	-1.4043e+00	-1.1857e+00	-8.3285e-01	-2.8064e-01	
4.5000e+01	-3.4185e+00	-3.6183e+00	-3.8646e+00	-4.1880e+00	
5.0000e+01	-4.5285e+00	-4.9339e+00	-5.4387e+00	-6.0796e+00	
5.5000e+01	-4.4165e+00	-4.7277e+00	-5.0821e+00	-5.4821e+00	
6.0000e+01	-3.6294e+00	-3.7585e+00	-3.8748e+00	-3.9632e+00	
6.5000e+01	-2.6946e+00	-2.6860e+00	-2.6441e+00	-2.5561e+00	
7.0000e+01	-1.8790e+00	-1.8018e+00	-1.6941e+00	-1.5505e+00	
7.5000e+01	-1.2578e+00	-1.1621e+00	-1.0461e+00	-9.0969e-01	
8.0000e+01	-8.1539e-01	-7.2821e-01	-6.3006e-01	-5.2314e-01	
8.5000e+01	-5.0841e-01	-4.4106e-01	-3.6887e-01	-2.9433e-01	
9.0000e+01	-2.9276e-01	-2.4840e-01	-2.0237e-01	-1.5656e-01	
9.5000e+01	-1.3237e-01	-1.1079e-01	-8.8802e-02	-6.7378e-02	
1.0000e+02	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	

Delta

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Stock Price	Time to Maturity (months)				
	5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00	
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0.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
5.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
1.0000e+01	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
1.5000e+01	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
2.0000e+01	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
2.5000e+01	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
3.0000e+01	-1.0000e+00	-1.0000e+00	-1.0000e+00	-1.0000e+00	
3.5000e+01	-9.8457e-01	-9.9042e-01	-9.9536e-01	-9.9883e-01	
4.0000e+01	-8.4152e-01	-8.5576e-01	-8.7084e-01	-8.8694e-01	
4.5000e+01	-6.0871e-01	-6.2173e-01	-6.3735e-01	-6.5654e-01	
5.0000e+01	-4.1584e-01	-4.2000e-01	-4.2463e-01	-4.2970e-01	
5.5000e+01	-2.6498e-01	-2.6123e-01	-2.5633e-01	-2.4982e-01	
6.0000e+01	-1.6069e-01	-1.5351e-01	-1.4500e-01	-1.3485e-01	
6.5000e+01	-9.4501e-02	-8.7208e-02	-7.9055e-02	-6.9969e-02	
7.0000e+01	-5.4694e-02	-4.8708e-02	-4.2358e-02	-3.5699e-02	
7.5000e+01	-3.1515e-02	-2.7084e-02	-2.2610e-02	-1.8177e-02	
8.0000e+01	-1.8291e-02	-1.5168e-02	-1.2164e-02	-9.3423e-03	
8.5000e+01	-1.0880e-02	-8.7026e-03	-6.7034e-03	-4.9214e-03	
9.0000e+01	-6.8562e-03	-5.2894e-03	-3.9132e-03	-2.7463e-03	
9.5000e+01	-4.8449e-03	-3.6251e-03	-2.5901e-03	-1.7469e-03	
1.0000e+02	-4.2199e-03	-3.1168e-03	-2.1936e-03	-1.4536e-03	

Gamma

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Stock Price	Time to Maturity (months)				
	5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00	
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0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
5.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
1.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
1.5000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
2.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
2.5000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
3.0000e+01	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	
3.5000e+01	6.1726e-03	3.8321e-03	1.8558e-03	4.6773e-04	
4.0000e+01	5.1047e-02	5.0031e-02	4.7953e-02	4.4288e-02	
4.5000e+01	4.2075e-02	4.3582e-02	4.5444e-02	4.7873e-02	
5.0000e+01	3.5072e-02	3.7109e-02	3.9646e-02	4.2863e-02	
5.5000e+01	2.5275e-02	2.6400e-02	2.7671e-02	2.9089e-02	
6.0000e+01	1.6442e-02	1.6688e-02	1.6860e-02	1.6900e-02	

6.5000e+01		1.0032e-02	9.8331e-03	9.5193e-03	9.0515e-03
7.0000e+01		5.8907e-03	5.5669e-03	5.1595e-03	4.6562e-03
7.5000e+01		3.3809e-03	3.0827e-03	2.7396e-03	2.3529e-03
8.0000e+01		1.9091e-03	1.6834e-03	1.4388e-03	1.1808e-03
8.5000e+01		1.0551e-03	9.0291e-04	7.4543e-04	5.8760e-04
9.0000e+01		5.5449e-04	4.6239e-04	3.7065e-04	2.8244e-04
9.5000e+01		2.5001e-04	2.0330e-04	1.5859e-04	1.1731e-04
1.0000e+02		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00

Lambda

Stock Price		Time to Maturity (months)			
		5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00
0.0000e+00		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
5.0000e+00		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
4.0000e+01		6.3243e+00	5.1893e+00	3.8089e+00	2.1118e+00
4.5000e+01		1.0721e+01	9.9718e+00	9.2140e+00	8.4953e+00
5.0000e+01		1.2381e+01	1.1807e+01	1.1228e+01	1.0636e+01
5.5000e+01		1.1483e+01	1.0837e+01	1.0142e+01	9.3795e+00
6.0000e+01		9.3227e+00	8.5840e+00	7.7870e+00	6.9211e+00
6.5000e+01		6.9621e+00	6.2206e+00	5.4412e+00	4.6264e+00
7.0000e+01		4.9268e+00	4.2651e+00	3.5937e+00	2.9227e+00
7.5000e+01		3.3602e+00	2.8204e+00	2.2920e+00	1.7866e+00
8.0000e+01		2.2221e+00	1.8126e+00	1.4248e+00	1.0683e+00
8.5000e+01		1.4122e+00	1.1240e+00	8.5856e-01	6.2248e-01
9.0000e+01		8.2686e-01	6.4587e-01	4.8252e-01	3.4083e-01
9.5000e+01		3.7891e-01	2.9252e-01	2.1553e-01	1.4976e-01
1.0000e+02		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00

Rho

Stock Price		Time to Maturity (months)			
		5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00
0.0000e+00		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
5.0000e+00		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
1.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.0000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3.5000e+01		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
4.0000e+01		-7.1918e+00	-6.0114e+00	-4.5204e+00	-2.5855e+00
4.5000e+01		-8.4541e+00	-7.6378e+00	-6.8479e+00	-6.1657e+00
5.0000e+01		-7.5988e+00	-6.9323e+00	-6.2879e+00	-5.6707e+00
5.5000e+01		-5.8905e+00	-5.2837e+00	-4.6809e+00	-4.0772e+00
6.0000e+01		-4.1854e+00	-3.6547e+00	-3.1306e+00	-2.6135e+00
6.5000e+01		-2.8221e+00	-2.3904e+00	-1.9743e+00	-1.5775e+00
7.0000e+01		-1.8437e+00	-1.5137e+00	-1.2055e+00	-9.2283e-01
7.5000e+01		-1.1812e+00	-9.4071e-01	-7.2326e-01	-5.3162e-01
8.0000e+01		-7.4513e-01	-5.7680e-01	-4.2921e-01	-3.0383e-01
8.5000e+01		-4.5907e-01	-3.4659e-01	-2.5060e-01	-1.7161e-01
9.0000e+01		-2.6550e-01	-1.9656e-01	-1.3892e-01	-9.2652e-02
9.5000e+01		-1.2280e-01	-8.9807e-02	-6.2569e-02	-4.1033e-02
1.0000e+02		0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00

