

# NAG Library Function Document

## nag\_ode\_ivp\_rkts\_interp (d02psc)

### 1 Purpose

nag\_ode\_ivp\_rkts\_interp (d02psc) computes the solution of a system of ordinary differential equations using interpolation anywhere on an integration step taken by nag\_ode\_ivp\_rkts\_onestep (d02pfc).

### 2 Specification

```
#include <nag.h>
#include <nagd02.h>

void nag_ode_ivp_rkts_interp (Integer n, double twant, Nag_SolDeriv request,
    Integer nwant, double ywant[], double ypwant[],
    void (*f)(double t, Integer n, const double y[], double yp[],
        Nag_Comm *comm),
    double wcomm[], Integer lwcomm, Nag_Comm *comm, Integer iwsav[],
    double rwsav[], NagError *fail)
```

### 3 Description

nag\_ode\_ivp\_rkts\_interp (d02psc) and its associated functions (nag\_ode\_ivp\_rkts\_onestep (d02pfc), nag\_ode\_ivp\_rkts\_setup (d02pqc), nag\_ode\_ivp\_rkts\_reset\_tend (d02prc), nag\_ode\_ivp\_rkts\_diag (d02ptc) and nag\_ode\_ivp\_rkts\_errass (d02puc)) solve the initial value problem for a first-order system of ordinary differential equations. The functions, based on Runge–Kutta methods and derived from RKSUITE (see Brankin *et al.* (1991)), integrate

$$y' = f(t, y) \quad \text{given} \quad y(t_0) = y_0$$

where  $y$  is the vector of  $n$  solution components and  $t$  is the independent variable.

nag\_ode\_ivp\_rkts\_onestep (d02pfc) computes the solution at the end of an integration step. Using the information computed on that step nag\_ode\_ivp\_rkts\_interp (d02psc) computes the solution by interpolation at any point on that step. It cannot be used if **method** = Nag\_RK\_7\_8 was specified in the call to setup function nag\_ode\_ivp\_rkts\_setup (d02pqc).

### 4 References

Brankin R W, Gladwell I and Shampine L F (1991) RKSUITE: A suite of Runge–Kutta codes for the initial value problems for ODEs *SoftReport 91-S1* Southern Methodist University

### 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:*  $n$ , the number of ordinary differential equations in the system to be solved by the integration function.  
*Constraint:*  $n \geq 1$ .
- 2: **twant** – double *Input*  
*On entry:*  $t$ , the value of the independent variable where a solution is desired.

- 3:     **request** – Nag\_SolDeriv *Input*  
*On entry:* determines whether the solution and/or its first derivative are to be computed  
**request** = Nag\_Sol  
    compute approximate solution.  
**request** = Nag\_Der  
    compute approximate first derivative.  
**request** = Nag\_SolDer  
    compute approximate solution and first derivative.  
*Constraint:* **request** = Nag\_Sol, Nag\_Der or Nag\_SolDer.
- 4:     **nwant** – Integer *Input*  
*On entry:* the number of components of the solution to be computed. The first **nwant** components are evaluated.  
*Constraint:*  $1 \leq \mathbf{nwant} \leq \mathbf{n}$ .
- 5:     **ywant**[**nwant**] – double *Output*  
*On exit:* an approximation to the first **nwant** components of the solution at **twant** if **request** = Nag\_Sol or Nag\_SolDer. Otherwise **ywant** is not defined.
- 6:     **ypwant**[**nwant**] – double *Output*  
*On exit:* an approximation to the first **nwant** components of the first derivative at **twant** if **request** = Nag\_Der or Nag\_SolDer. Otherwise **ypwant** is not defined.
- 7:     **f** – function, supplied by the user *External Function*  
**f** must evaluate the functions  $f_i$  (that is the first derivatives  $y'_i$ ) for given values of the arguments  $t, y_i$ . It must be the same procedure as supplied to nag\_ode\_ivp\_rkts\_onestep (d02pfc).

The specification of **f** is:

```
void f (double t, Integer n, const double y[], double yp[],
       Nag_Comm *comm)
```

- 1:     **t** – double *Input*  
*On entry:*  $t$ , the current value of the independent variable.
- 2:     **n** – Integer *Input*  
*On entry:*  $n$ , the number of ordinary differential equations in the system to be solved.
- 3:     **y**[**n**] – const double *Input*  
*On entry:* the current values of the dependent variables,  $y_i$ , for  $i = 1, 2, \dots, n$ .
- 4:     **yp**[**n**] – double *Output*  
*On exit:* the values of  $f_i$ , for  $i = 1, 2, \dots, n$ .
- 5:     **comm** – Nag\_Comm \*  
    Pointer to structure of type Nag\_Comm; the following members are relevant to **f**.

**user** – double \*  
**iuser** – Integer \*  
**p** – Pointer

The type Pointer will be `void *`. Before calling `nag_ode_ivp_rkts_interp` (d02psc) you may allocate memory and initialize these pointers with various quantities for use by **f** when called from `nag_ode_ivp_rkts_interp` (d02psc) (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

- 8: **wcomm**[**lwcomm**] – double *Communication Array*  
*On entry*: this array stores information that can be utilized on subsequent calls to `nag_ode_ivp_rkts_interp` (d02psc).
- 9: **lwcomm** – Integer *Input*  
*On entry*: length of **wcomm**.  
 If in a previous call to `nag_ode_ivp_rkts_setup` (d02pqc):  
     **method** = Nag\_RK\_2\_3 then **lwcomm** must be at least 1.  
     **method** = Nag\_RK\_4\_5 then **lwcomm** must be at least  $n + \max(n, 5 \times \text{nwant})$ .  
     **method** = Nag\_RK\_7\_8 then **wcomm** and **lwcomm** are not referenced.
- 10: **comm** – Nag\_Comm \*  
 The NAG communication argument (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).
- 11: **iwsav**[130] – Integer *Communication Array*  
 12: **rwsav**[ $32 \times n + 350$ ] – double *Communication Array*  
*On entry*: these must be the same arrays supplied in a previous call `nag_ode_ivp_rkts_onestep` (d02pfc). They must remain unchanged between calls.  
*On exit*: information about the integration for use on subsequent calls to `nag_ode_ivp_rkts_one step` (d02pfc), `nag_ode_ivp_rkts_interp` (d02psc) or other associated functions.
- 13: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle \text{value} \rangle$  had an illegal value.

### NE\_INT

On entry, **lwcomm** =  $\langle \text{value} \rangle$ .

Constraint: for **method** = Nag\_RK\_2\_3, **lwcomm**  $\geq 1$ .

**NE\_INT\_2**

On entry, **nwant** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint:  $1 \leq \mathbf{nwant} \leq \mathbf{n}$ .

**NE\_INT\_3**

On entry, **lwcomm** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$  and **nwant** =  $\langle value \rangle$ .  
 Constraint: for **method** = Nag\_RK\_4\_5,  $\mathbf{lwcomm} \geq \mathbf{n} + \max(\mathbf{n}, 5 \times \mathbf{nwant})$ .

**NE\_INT\_CHANGED**

On entry, **n** =  $\langle value \rangle$ , but the value passed to the setup function was **n** =  $\langle value \rangle$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_MISSING\_CALL**

You cannot call this function before you have called the step integrator.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_PREV\_CALL**

On entry, a previous call to the setup function has not been made or the communication arrays have become corrupted, or a catastrophic error has already been detected elsewhere.  
 You cannot continue integrating the problem.

**NE\_PREV\_CALL\_INI**

You cannot call this function after the integrator has returned an error.

**NE\_RK\_INVALID\_CALL**

You cannot call this function when you have specified, in the setup function, that the range integrator will be used.

**NE\_RK\_NO\_INTERP**

**method** = Nag\_RK\_7\_8 in setup, but interpolation is not available for this method. Either use **method** = Nag\_RK\_4\_5 in setup or use reset function to force the integrator to step to particular points.

**7 Accuracy**

The computed values will be of a similar accuracy to that computed by nag\_ode\_ivp\_rkts\_onestep (d02pfc).

**8 Parallelism and Performance**

nag\_ode\_ivp\_rkts\_interp (d02psc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

None.

## 10 Example

This example solves the equation

$$y'' = -y, \quad y(0) = 0, \quad y'(0) = 1$$

reposed as

$$y'_1 = y_2$$

$$y'_2 = -y_1$$

over the range  $[0, 2\pi]$  with initial conditions  $y_1 = 0.0$  and  $y_2 = 1.0$ . Relative error control is used with threshold values of  $1.0\text{e-}8$  for each solution component. `nag_ode_ivp_rkts_onestep` (d02pfc) is used to integrate the problem one step at a time and `nag_ode_ivp_rkts_interp` (d02psc) is used to compute the first component of the solution and its derivative at intervals of length  $\pi/8$  across the range whenever these points lie in one of those integration steps. A low order Runge–Kutta method (`method` = `Nag_RK_2_3`) is also used with tolerances `tol` =  $1.0\text{e-}4$  and `tol` =  $1.0\text{e-}5$  in turn so that solutions may be compared.

### 10.1 Program Text

```
/* nag_ode_ivp_rkts_interp (d02psc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd02.h>

#ifdef __cplusplus
extern "C"
{
#endif
    static void NAG_CALL f(double t, Integer n, const double *y,
                           double *yp, Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

#define N 2

int main(void)
{
    /* Scalars */
    double tol0 = 1.0e-3;
    Integer npts = 16, exit_status = 0;
    Integer liwsav, lrwsav, lwcomm, n;
    double hnext, hstart, tend, tgot, tinc, tol, tstart, twant, waste;
    Integer fevals, i, j, k, stepcost, stepsok;
    /* Arrays */
    static double ruser[1] = { -1.0 };
    double *rwsav = 0, *thresh = 0, *ygot = 0, *yinit = 0, *ypgot = 0;
```

```

double *ywant = 0, *ypwant = 0, *wcomm = 0;
Integer *iwsav = 0;
char nag_enum_arg[40];
/* NAG types */
NagError fail;
Nag_RK_method method;
Nag_ErrorAssess errass;
Nag_SolDeriv request = Nag_SolDer;
Nag_Comm comm;

INIT_FAIL(fail);

n = N;
liwsav = 130;
lrwsav = 350 + 32 * n;
lwcomm = 6 * n;
printf("nag_ode_ivp_rkts_interp (d02psc) Example Program Results\n\n");

/* For communication with user-supplied functions: */
comm.user = ruser;

if (!(thresh = NAG_ALLOC(n, double)) ||
    !(ygot = NAG_ALLOC(n, double)) ||
    !(yinit = NAG_ALLOC(n, double)) ||
    !(ypgot = NAG_ALLOC(n, double)) ||
    !(ywant = NAG_ALLOC(n, double)) ||
    !(ypwant = NAG_ALLOC(n, double)) ||
    !(iwsav = NAG_ALLOC(liwsav, Integer)) ||
    !(rwsav = NAG_ALLOC(lrwsav, double)) ||
    !(wcomm = NAG_ALLOC(lwcomm, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Set initial conditions for ODE and parameters for the integrator. */

#ifdef _WIN32
    scanf_s(" %39s%*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n] ", nag_enum_arg);
#endif
/* nag_enum_name_to_value (x04nac) Converts NAG enum member name to value. */
method = (Nag_RK_method) nag_enum_name_to_value(nag_enum_arg);

#ifdef _WIN32
    scanf_s(" %39s%*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n] ", nag_enum_arg);
#endif
errass = (Nag_ErrorAssess) nag_enum_name_to_value(nag_enum_arg);

#ifdef _WIN32
    scanf_s("%lf%lf%*[\n] ", &tstart, &tend);
#else
    scanf("%lf%lf%*[\n] ", &tstart, &tend);
#endif

for (j = 0; j < n; j++)
#ifdef _WIN32
    scanf_s("%lf", &yinit[j]);
#else
    scanf("%lf", &yinit[j]);

```

```

#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%lf%*[\n] ", &hstart);
#else
    scanf("%lf%*[\n] ", &hstart);
#endif
    for (j = 0; j < n; j++)
#ifdef _WIN32
        scanf_s("%lf", &thresh[j]);
#else
        scanf("%lf", &thresh[j]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    tinc = (tend - tstart) / (double) (npts);
    tol = tol0;
    for (i = 1; i <= 2; i++) {
        tol = tol * 0.1;
        /* Initialize Runge-Kutta method for integrating ODE using
         * nag_ode_ivp_rkts_setup (d02pqc).
         */
        nag_ode_ivp_rkts_setup(n, tstart, tend, yinit, tol, thresh, method,
                               errass, hstart, iwsav, rwsav, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_ode_ivp_rkts_setup (d02pqc).\n%s\n",
                   fail.message);
            exit_status = 1;
            goto END;
        }

        printf(" Calculation with tol = %8.1e\n", tol);
        printf("      t          y1          y1'\n");
        printf("%6.3f", tstart);
        for (k = 0; k < n; k++)
            printf("    %8.4f", yinit[k]);
        printf("\n");

        /* Set up first point at which solution is desired. */
        twant = tstart + tinc;
        tgot = tstart;
        /* Integrate by single steps until tend is reached or error is
         * encountered. Solution is required at regular increments, requiring
         * interpolation on those steps that pass over the regular grid values
         * of t.
         */
        while (tgot < tend) {
            /* Solve ODE by Runge-Kutta method by a sequence of single steps using
             * nag_ode_ivp_rkts_onestep (d02pfc).
             */
            nag_ode_ivp_rkts_onestep(f, n, &tgot, ygot, ypgot, &comm,
                                     iwsav, rwsav, &fail);
            if (fail.code != NE_NOERROR) {
                printf("Error from nag_ode_ivp_rkts_onestep (d02pfc).\n%s\n",
                       fail.message);
                exit_status = 2;
                goto END;
            }
        }

        /* Interpolate onto those grid values passed over in by last step. */
        while (twant <= tgot) {
            /* Interpolate at t = twant, given solution by

```

```

    * nag_ode_ivp_rkts_onestep (d02pfc), using
    * nag_ode_ivp_rkts_interp (d02psc).
    */
    nag_ode_ivp_rkts_interp(n, twant, request, n, ywant, ypwant, f,
                           wcomm, lwcomm, &comm, iwsav, rwsav, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_ivp_rkts_interp (d02psc).\n%s\n",
              fail.message);
        exit_status = 3;
        goto END;
    }
    printf("%6.3f    %8.4f    %8.4f\n", twant, ywant[0], ypwant[0]);
    /* Set next required solution point. */
    twant = twant + tinc;
}
}
/* Get diagnostics on whole integration using
 * nag_ode_ivp_rkts_diag (d02ptc).
 */
nag_ode_ivp_rkts_diag(&fevals, &stepcost, &waste, &stepsok, &hnext,
                     iwsav, rwsav, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ode_ivp_rkts_diag (d02ptc).\n%s\n",
          fail.message);
    exit_status = 4;
    goto END;
}
printf("Cost of the integration in evaluations of f is %6" NAG_IFMT
      "\n\n", fevals);
}
END:
NAG_FREE(thresh);
NAG_FREE(yinit);
NAG_FREE(ygot);
NAG_FREE(ypgot);
NAG_FREE(ywant);
NAG_FREE(ypwant);
NAG_FREE(rwsav);
NAG_FREE(iwsav);
NAG_FREE(wcomm);
return exit_status;
}

static void NAG_CALL f(double t, Integer n, const double *y, double *yp,
                      Nag_Comm *comm)
{
    if (comm->user[0] == -1.0) {
        printf("(User-supplied callback f, first invocation.)\n");
        comm->user[0] = 0.0;
    }
    yp[0] = y[1];
    yp[1] = -y[0];
}

```

## 10.2 Program Data

nag\_ode\_ivp\_rkts\_interp (d02psc) Example Program Data

Nag_RK_2_3	:	method
Nag_ErrorAssess_off	:	errass
0.0      6.28318530717958647692	:	tstart, tend
0.0      1.0	:	yinit(1:n)
0.0	:	hstart
1.0E-8   1.0E-8	:	thresh(1:n)



### 10.3 Program Results

nag\_ode\_ivp\_rkts\_interp (d02psc) Example Program Results

```

Calculation with tol = 1.0e-04
  t      y1      y1'
0.000    0.0000    1.0000
(User-supplied callback f, first invocation.)
0.393    0.3827    0.9238
0.785    0.7071    0.7070
1.178    0.9238    0.3826
1.571    0.9999   -0.0000
1.963    0.9238   -0.3827
2.356    0.7070   -0.7071
2.749    0.3826   -0.9238
3.142   -0.0000   -0.9998
3.534   -0.3826   -0.9237
3.927   -0.7070   -0.7069
4.320   -0.9237   -0.3826
4.712   -0.9998    0.0000
5.105   -0.9236    0.3827
5.498   -0.7069    0.7070
5.890   -0.3825    0.9236
Cost of the integration in evaluations of f is      235

```

```

Calculation with tol = 1.0e-05
  t      y1      y1'
0.000    0.0000    1.0000
0.393    0.3827    0.9239
0.785    0.7071    0.7071
1.178    0.9239    0.3827
1.571    1.0000   -0.0000
1.963    0.9239   -0.3827
2.356    0.7071   -0.7071
2.749    0.3827   -0.9239
3.142   -0.0000   -1.0000
3.534   -0.3827   -0.9239
3.927   -0.7071   -0.7071
4.320   -0.9239   -0.3827
4.712   -1.0000    0.0000
5.105   -0.9239    0.3827
5.498   -0.7071    0.7071
5.890   -0.3827    0.9239
Cost of the integration in evaluations of f is      493

```

