

NAG Library Function Document

nag_sum_fft_real_3d (c06pyc)

1 Purpose

nag_sum_fft_real_3d (c06pyc) computes the three-dimensional discrete Fourier transform of a trivariate sequence of real data values.

2 Specification

```
#include <nag.h>
#include <nagc06.h>

void nag_sum_fft_real_3d (Integer n1, Integer n2, Integer n3,
    const double x[], Complex y[], NagError *fail)
```

3 Description

nag_sum_fft_real_3d (c06pyc) computes the three-dimensional discrete Fourier transform of a trivariate sequence of real data values $x_{j_1 j_2 j_3}$, for $j_1 = 0, 1, \dots, n_1 - 1$, $j_2 = 0, 1, \dots, n_2 - 1$ and $j_3 = 0, 1, \dots, n_3 - 1$.

The discrete Fourier transform is here defined by

$$\hat{z}_{k_1 k_2 k_3} = \frac{1}{\sqrt{n_1 n_2 n_3}} \sum_{j_1=0}^{n_1-1} \sum_{j_2=0}^{n_2-1} \sum_{j_3=0}^{n_3-1} x_{j_1 j_2 j_3} \times \exp \left(-2\pi i \left(\frac{j_1 k_1}{n_1} + \frac{j_2 k_2}{n_2} + \frac{j_3 k_3}{n_3} \right) \right),$$

where $k_1 = 0, 1, \dots, n_1 - 1$, $k_2 = 0, 1, \dots, n_2 - 1$ and $k_3 = 0, 1, \dots, n_3 - 1$. (Note the scale factor of $\frac{1}{\sqrt{n_1 n_2 n_3}}$ in this definition.)

The transformed values $\hat{z}_{k_1 k_2 k_3}$ are complex. Because of conjugate symmetry (i.e., $\hat{z}_{k_1 k_2 k_3}$ is the complex conjugate of $\hat{z}_{(n_1-k_1)(n_2-k_2)(n_3-k_3)}$), only slightly more than half of the Fourier coefficients need to be stored in the output.

A call of nag_sum_fft_real_3d (c06pyc) followed by a call of nag_sum_fft_hermitian_3d (c06pzc) will restore the original data.

This function performs multiple one-dimensional discrete Fourier transforms by the fast Fourier transform (FFT) algorithm in Brigham (1974) and Temperton (1983).

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

Temperton C (1983) Fast mixed-radix real Fourier transforms *J. Comput. Phys.* **52** 340–350

5 Arguments

- | | | |
|----|---|--------------|
| 1: | n1 – Integer | <i>Input</i> |
| | <i>On entry:</i> n_1 , the first dimension of the transform. | |
| | <i>Constraint:</i> $n1 \geq 1$. | |
| 2: | n2 – Integer | <i>Input</i> |
| | <i>On entry:</i> n_2 , the second dimension of the transform. | |
| | <i>Constraint:</i> $n2 \geq 1$. | |

- 3: **n3** – Integer *Input*
On entry: n_3 , the third dimension of the transform.
Constraint: $\mathbf{n3} \geq 1$.
- 4: **x[n1 × n2 × n3]** – const double *Input*
On entry: the real input dataset x , where $x_{j_1 j_2 j_3}$ is stored in $\mathbf{x}[j_3 \times n_1 n_2 + j_2 \times n_1 + j_1]$, for $j_1 = 0, 1, \dots, n_1 - 1$, $j_2 = 0, 1, \dots, n_2 - 1$ and $j_3 = 0, 1, \dots, n_3 - 1$.
- 5: **y[dim]** – Complex *Output*
Note: the dimension, dim , of the array **y** must be at least $(\mathbf{n1}/2 + 1) \times \mathbf{n2} \times \mathbf{n3}$.
On exit: the complex output dataset \hat{z} , where $\hat{z}_{k_1 k_2 k_3}$ is stored in $\mathbf{y}[k_3 \times (n_1/2 + 1)n_2 + k_2 \times (n_1/2 + 1) + k_1]$, for $k_1 = 0, 1, \dots, n_1/2$, $k_2 = 0, 1, \dots, n_2 - 1$ and $k_3 = 0, 1, \dots, n_3 - 1$. Note the first dimension is cut roughly by half to remove the redundant information due to conjugate symmetry.
- 6: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n1} = \langle value \rangle$.

Constraint: $\mathbf{n1} \geq 1$.

On entry, $\mathbf{n2} = \langle value \rangle$.

Constraint: $\mathbf{n2} \geq 1$.

On entry, $\mathbf{n3} = \langle value \rangle$.

Constraint: $\mathbf{n3} \geq 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Some indication of accuracy can be obtained by performing a forward transform using `nag_sum_fft_real_3d` (c06pyc) and a backward transform using `nag_sum_fft_hermitian_3d` (c06pzc), and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Parallelism and Performance

`nag_sum_fft_real_3d` (c06pyc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_sum_fft_real_3d` (c06pyc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The time taken by `nag_sum_fft_real_3d` (c06pyc) is approximately proportional to $n_1 n_2 n_3 \log(n_1 n_2 n_3)$, but also depends on the factors of n_1 , n_2 and n_3 . `nag_sum_fft_real_3d` (c06pyc) is fastest if the only prime factors of n_1 , n_2 and n_3 are 2, 3 and 5, and is particularly slow if one of the dimensions is a large prime, or has large prime factors.

Workspace is internally allocated by `nag_sum_fft_real_3d` (c06pyc). The total size of these arrays is approximately proportional to $n_1 n_2 n_3$.

10 Example

This example reads in a trivariate sequence of real data values and prints their discrete Fourier transforms as computed by `nag_sum_fft_real_3d` (c06pyc). Inverse transforms are then calculated by calling `nag_sum_fft_hermitian_3d` (c06pzc) showing that the original sequences are restored.

10.1 Program Text

```
/* nag_sum_fft_real_3d (c06pyc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagc06.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, k, n1, n2, n3;
    /* Arrays */
    Complex *y = 0;
    double *x = 0;
    char title[30];
    /* Nag Types */
    NagError fail;

    INIT_FAIL(fail);
```

```

printf("nag_sum_fft_real_3d (c06pyc) Example Program Results\n");
fflush(stdout);

/* Read dimensions of array from data file. */
#ifdef _WIN32
scanf_s("%*[\n] %" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n1, &n2,
        &n3);
#else
scanf("%*[\n] %" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n1, &n2,
        &n3);
#endif

if (!(x = NAG_ALLOC(n1 * n2 * n3, double)) ||
    !(y = NAG_ALLOC((n1 / 2 + 1) * n2 * n3, Complex)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read array values from data file and print out. */
for (k = 0; k < n1 * n2 * n3; k++)
#ifdef _WIN32
    scanf_s("%lf", &x[k]);
#else
    scanf("%lf", &x[k]);
#endif

printf("\nBelow we define X(i,j,k)=x[k*n1*n2+j*n1+i]");
printf(" where i and j are the row and column \n");
printf("indices of the matrices printed.");
printf(" Y is defined similarly (but having n1/2+1 rows\n");
printf("only due to conjugate symmetry).\n");

printf("\n Original data values\n");
fflush(stdout);
for (k = 0; k < n3; k++) {
#ifdef _WIN32
    sprintf_s(title, (unsigned)_countof(title),
        "\n X(i,j,k) for k = %" NAG_IFMT, k);
#else
    sprintf(title, "\n X(i,j,k) for k = %" NAG_IFMT, k);
#endif
    nag_gen_real_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                Nag_NonUnitDiag, n1, n2, &x[k * n1 * n2], n1,
                                "%6.3f", title, Nag_NoLabels, 0,
                                Nag_NoLabels, 0, 80, 0, 0, &fail);
}
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print_comp (x04cbc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

/* Compute three-dimensional real-to-complex discrete Fourier transform using
 * nag_sum_fft_real_3d (c06pyc) and print out.
 */
nag_sum_fft_real_3d(n1, n2, n3, x, y, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_sum_fft_real_3d (c06pyc).\n%s\n", fail.message);
    exit_status = 2;
    goto END;
}

printf("\n Components of discrete Fourier transform\n");
fflush(stdout);
for (k = 0; k < n3; k++) {
#ifdef _WIN32
    sprintf_s(title, (unsigned)_countof(title),
        "\n Y(i,j,k) for k = %" NAG_IFMT, k);

```

```

#else
    sprintf(title, "\n Y(i,j,k) for k = %" NAG_IFMT, k);
#endif
    /* nag_gen_complx_mat_print_comp (x04dbc).
    * Print complex general matrix (comprehensive) */
    nag_gen_complx_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                  Nag_NonUnitDiag, n1 / 2 + 1, n2,
                                  &y[k * (n1 / 2 + 1) * n2], n1 / 2 + 1,
                                  Nag_BracketForm, "%6.3f", title,
                                  Nag_NoLabels, 0, Nag_NoLabels, 0, 90, 0, 0,
                                  &fail);
}
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
          fail.message);
    exit_status = 3;
    goto END;
}

/* Compute three-dimensional complex-to-real discrete Fourier transform using
* nag_sum_fft_hermitian_3d (c06pzc) and print out.
*/
nag_sum_fft_hermitian_3d(n1, n2, n3, y, x, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_sum_fft_hermitian_3d (c06pzc).\n%s\n",
          fail.message);
    exit_status = 4;
    goto END;
}

printf("\n Original sequence as restored by inverse transform\n");
fflush(stdout);
for (k = 0; k < n3; k++) {
#ifdef _WIN32
    sprintf_s(title, (unsigned)_countof(title),
              "\n X(i,j,k) for k = %" NAG_IFMT, k);
#else
    sprintf(title, "\n X(i,j,k) for k = %" NAG_IFMT, k);
#endif
    /* nag_gen_real_mat_print_comp (x04cbc).
    * Print out a real matrix (comprehensive) */
    nag_gen_real_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                Nag_NonUnitDiag, n1, n2, &x[k * n1 * n2], n1,
                                "%6.3f", title, Nag_NoLabels, 0,
                                Nag_NoLabels, 0, 80, 0, 0, &fail);
}
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print_comp (x04cbc).\n%s\n",
          fail.message);
    exit_status = 5;
    goto END;
}

END:
    NAG_FREE(x);
    NAG_FREE(y);
    return exit_status;
}

```

10.2 Program Data

nag_sum_fft_real_3d (c06pyc) Example Program Data

3	3	4	: n1, n2, n3
1.541	0.584	0.010	
0.346	1.284	1.960	
1.754	0.855	0.089	
0.161	1.004	1.844	

1.907	1.137	0.240
0.042	0.725	1.660
1.989	1.408	0.452
0.001	0.467	1.424
1.991	1.647	0.708
0.037	0.252	1.154
1.915	1.834	0.987
0.151	0.096	0.872 : x

10.3 Program Results

nag_sum_fft_real_3d (c06pyc) Example Program Results

Below we define $X(i,j,k)=x[k*n1*n2+j*n1+i]$ where i and j are the row and column indices of the matrices printed. Y is defined similarly (but having $n1/2+1$ rows only due to conjugate symmetry).

Original data values

$X(i,j,k)$ for $k = 0$
 1.541 0.346 1.754
 0.584 1.284 0.855
 0.010 1.960 0.089

$X(i,j,k)$ for $k = 1$
 0.161 1.907 0.042
 1.004 1.137 0.725
 1.844 0.240 1.660

$X(i,j,k)$ for $k = 2$
 1.989 0.001 1.991
 1.408 0.467 1.647
 0.452 1.424 0.708

$X(i,j,k)$ for $k = 3$
 0.037 1.915 0.151
 0.252 1.834 0.096
 1.154 0.987 0.872

Components of discrete Fourier transform

$Y(i,j,k)$ for $k = 0$
 (5.755, 0.000) (-0.268,-0.420) (-0.268, 0.420)
 (0.081, 0.015) (0.038, 0.198) (0.067,-0.122)

$Y(i,j,k)$ for $k = 1$
 (-0.277,-0.237) (0.109,-0.756) (-0.688, 0.210)
 (0.060, 0.156) (-0.275, 0.295) (0.280, 0.012)

$Y(i,j,k)$ for $k = 2$
 (0.415, 0.000) (0.175, 0.871) (0.175,-0.871)
 (0.645,-0.478) (1.585, 0.616) (-0.113,-1.555)

$Y(i,j,k)$ for $k = 3$
 (-0.277, 0.237) (-0.688,-0.210) (0.109, 0.756)
 (0.047,-0.077) (0.201, 0.061) (-0.128,-0.117)

Original sequence as restored by inverse transform

$X(i,j,k)$ for $k = 0$
 1.541 0.346 1.754
 0.584 1.284 0.855
 0.010 1.960 0.089

$X(i,j,k)$ for $k = 1$
 0.161 1.907 0.042
 1.004 1.137 0.725
 1.844 0.240 1.660

```
X(i,j,k) for k = 2  
1.989  0.001  1.991  
1.408  0.467  1.647  
0.452  1.424  0.708
```

```
X(i,j,k) for k = 3  
0.037  1.915  0.151  
0.252  1.834  0.096  
1.154  0.987  0.872
```
