

NAG Library Function Document

nag_fft_multiple_real (c06fpc)

1 Purpose

`nag_fft_multiple_real` (c06fpc) computes the discrete Fourier transforms of m sequences, each containing n real data values.

2 Specification

```
#include <nag.h>
#include <nagc06.h>

void nag_fft_multiple_real (Integer m, Integer n, double x[],
    const double trig[], NagError *fail)
```

3 Description

Given m sequences of n real data values x_j^p , for $j = 0, 1, \dots, n-1$ and $p = 1, 2, \dots, m$, this function simultaneously calculates the Fourier transforms of all the sequences defined by

$$\hat{z}_k^p = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j^p \exp(-2\pi i j k / n), \quad \text{for } k = 0, 1, \dots, n-1; p = 1, 2, \dots, m.$$

(Note the scale factor $1/\sqrt{n}$ in this definition.)

The transformed values \hat{z}_k^p are complex, but for each value of p the \hat{z}_k^p form a Hermitian sequence (i.e., \hat{z}_{n-k}^p is the complex conjugate of \hat{z}_k^p), so they are completely determined by mn real numbers. The first call of `nag_fft_multiple_real` (c06fpc) must be preceded by a call to `nag_fft_init_trig` (c06gzc) to initialize the array `trig` with trigonometric coefficients according to the value of `n`.

The discrete Fourier transform is sometimes defined using a positive sign in the exponential term

$$\hat{z}_k^p = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j^p \exp(+2\pi i j k / n).$$

To compute this form, this function should be followed by a call to `nag_multiple_conjugate_hermitian` (c06gqc) to form the complex conjugates of the \hat{z}_k^p .

The function uses a variant of the fast Fourier transform algorithm (Brigham (1974)) known as the Stockham self-sorting algorithm, which is described in Temperton (1983). Special coding is provided for the factors 2, 3, 4, 5 and 6.

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

Temperton C (1983) Fast mixed-radix real Fourier transforms *J. Comput. Phys.* **52** 340–350

5 Arguments

- 1: **m** – Integer *Input*
On entry: the number of sequences to be transformed, m .
Constraint: **m** ≥ 1 .

- 2: **n** – Integer *Input*
On entry: the number of real values in each sequence, n .
Constraint: $n \geq 1$.
- 3: **x**[**m** × **n**] – double *Input/Output*
On entry: the m data sequences must be stored in **x** consecutively. If the data values of the p th sequence to be transformed are denoted by x_j^p , for $j = 0, 1, \dots, n-1$, then the mn elements of the array **x** must contain the values
- $$x_0^1, x_1^1, \dots, x_{n-1}^1, x_0^2, x_1^2, \dots, x_{n-1}^2, \dots, x_0^m, x_1^m, \dots, x_{n-1}^m.$$
- On exit:* the m discrete Fourier transforms in Hermitian form, stored consecutively, overwriting the corresponding original sequences. If the n components of the discrete Fourier transform \hat{z}_k^p are written as $d_k^p + ib_k^p$, then for $0 \leq k \leq n/2$, d_k^p is in array element **x**[($p-1$) × $n + k$] and for $1 \leq k \leq (n-1)/2$, b_k^p is in array element **x**[($p-1$) × $n + n - k$].
- 4: **trig**[2 × **n**] – const double *Input*
On entry: trigonometric coefficients as returned by a call of nag_fft_init_trig (c06gzc). nag_fft_multiple_real (c06fpc) makes a simple check to ensure that **trig** has been initialized and that the initialization is compatible with the value of **n**.
- 5: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_C06_NOT_TRIG

Value of **n** and **trig** array are incompatible or **trig** array not initialized.

NE_INT_ARG_LT

On entry, **m** = $\langle value \rangle$.

Constraint: $m \geq 1$.

On entry, **n** = $\langle value \rangle$.

Constraint: $n \geq 1$.

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Parallelism and Performance

nag_fft_multiple_real (c06fpc) is not threaded in any implementation.

9 Further Comments

The time taken is approximately proportional to $nm \log(n)$, but also depends on the factors of n . The function is fastest if the only prime factors of n are 2, 3 and 5, and is particularly slow if n is a large prime, or has large prime factors.

10 Example

This program reads in sequences of real data values and prints their discrete Fourier transforms (as computed by `nag_fft_multiple_real` (c06fpc)). The Fourier transforms are expanded into full complex form using `nag_multiple_hermitian_to_complex` (c06gsc) and printed. Inverse transforms are then calculated by calling `nag_multiple_conjugate_hermitian` (c06ggc) followed by `nag_fft_multiple_hermitian` (c06fqc) showing that the original sequences are restored.

10.1 Program Text

```
/* nag_fft_multiple_real (c06fpc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagc06.h>

int main(void)
{
    Integer exit_status = 0, i, j, m, n;
    NagError fail;
    double *trig = 0, *u = 0, *v = 0, *x = 0;

    INIT_FAIL(fail);

    printf("nag_fft_multiple_real (c06fpc) Example Program Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    while (scanf_s("%" NAG_IFMT "%" NAG_IFMT "", &m, &n) != EOF)
#else
    while (scanf("%" NAG_IFMT "%" NAG_IFMT "", &m, &n) != EOF)
#endif
    {
        if (m >= 1 && n >= 1) {
            if (!(trig = NAG_ALLOC(2 * n, double)) ||
                !(u = NAG_ALLOC(m * n, double)) ||
                !(v = NAG_ALLOC(m * n, double)) || !(x = NAG_ALLOC(m * n, double)))
            {
                printf("Allocation failure\n");
                exit_status = -1;
                goto END;
            }
        }
        else {
            printf("Invalid m or n.\n");
            exit_status = 1;
            return exit_status;
        }

        printf("\n\nm = %2" NAG_IFMT " n = %2" NAG_IFMT "\n", m, n);
        /* Read in data and print out. */
        for (j = 0; j < m; ++j)
            for (i = 0; i < n; ++i)
#ifdef _WIN32
                scanf_s("%lf", &x[j * n + i]);
#else
                scanf("%lf", &x[j * n + i]);
#endif
    }
}
```

```

#endif
printf("\nOriginal data values\n\n");
for (j = 0; j < m; ++j) {
    printf("    ");
    for (i = 0; i < n; ++i)
        printf("%10.4f%s", x[j * n + i],
            (i % 6 == 5 && i != n - 1 ? "\n    " : ""));
    printf("\n");
}
/* nag_fft_init_trig (c06gzc).
 * Initialization function for other c06 functions
 */
nag_fft_init_trig(n, trig, &fail); /* Initialize trig array */
if (fail.code != NE_NOERROR) {
    printf("Error from nag_fft_init_trig (c06gzc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Calculate transforms */
/* nag_fft_multiple_real (c06fpc).
 * Multiple one-dimensional real discrete Fourier transforms
 */
nag_fft_multiple_real(m, n, x, trig, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_fft_multiple_real (c06fpc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}
printf("\nDiscrete Fourier transforms in Hermitian format\n\n");
for (j = 0; j < m; ++j) {
    printf("    ");
    for (i = 0; i < n; ++i)
        printf("%10.4f%s", x[j * n + i],
            (i % 6 == 5 && i != n - 1 ? "\n    " : ""));
    printf("\n");
}
/* Calculate full complex form of Hermitian result */
/* nag_multiple_hermitian_to_complex (c06gsc).
 * Convert Hermitian sequences to general complex sequences
 */
nag_multiple_hermitian_to_complex(m, n, x, u, v, &fail);
printf("\nFourier transforms in full complex form\n\n");
for (j = 0; j < m; ++j) {
    printf("Real");
    for (i = 0; i < n; ++i)
        printf("%10.4f%s", u[j * n + i],
            (i % 6 == 5 && i != n - 1 ? "\n    " : ""));
    printf("\nImag");
    for (i = 0; i < n; ++i)
        printf("%10.4f%s", v[j * n + i],
            (i % 6 == 5 && i != n - 1 ? "\n    " : ""));
    printf("\n\n");
}
/* Calculate inverse transforms */
/* Conjugate Hermitian sequences of transforms */
/* nag_multiple_conjugate_hermitian (c06gqc).
 * Complex conjugate of multiple Hermitian sequences
 */
nag_multiple_conjugate_hermitian(m, n, x, &fail);
/* Transform to give inverse transforms */
/* nag_fft_multiple_hermitian (c06fqc).
 * Multiple one-dimensional Hermitian discrete Fourier
 * transforms
 */
nag_fft_multiple_hermitian(m, n, x, trig, &fail);
printf("\nOriginal data as restored by inverse transform\n\n");
for (j = 0; j < m; ++j) {
    printf("    ");
    for (i = 0; i < n; ++i)
        printf("%10.4f%s", x[j * n + i],

```

```

        (i % 6 == 5 && i != n - 1 ? "\n      " : "");
    printf("\n");
}
END:
    NAG_FREE(trig);
    NAG_FREE(u);
    NAG_FREE(v);
    NAG_FREE(x);
}
return exit_status;
}

```

10.2 Program Data

nag_fft_multiple_real (c06fpc) Example Program Data

3	6				
0.3854	0.6772	0.1138	0.6751	0.6362	0.1424
0.5417	0.2983	0.1181	0.7255	0.8638	0.8723
0.9172	0.0644	0.6037	0.6430	0.0428	0.4815

10.3 Program Results

nag_fft_multiple_real (c06fpc) Example Program Results

m = 3 n = 6

Original data values

0.3854	0.6772	0.1138	0.6751	0.6362	0.1424
0.5417	0.2983	0.1181	0.7255	0.8638	0.8723
0.9172	0.0644	0.6037	0.6430	0.0428	0.4815

Discrete Fourier transforms in Hermitian format

1.0737	-0.1041	0.1126	-0.1467	-0.3738	-0.0044
1.3961	-0.0365	0.0780	-0.1521	-0.0607	0.4666
1.1237	0.0914	0.3936	0.1530	0.3458	-0.0508

Fourier transforms in full complex form

Real	1.0737	-0.1041	0.1126	-0.1467	0.1126	-0.1041
Imag	0.0000	-0.0044	-0.3738	0.0000	0.3738	0.0044
Real	1.3961	-0.0365	0.0780	-0.1521	0.0780	-0.0365
Imag	0.0000	0.4666	-0.0607	0.0000	0.0607	-0.4666
Real	1.1237	0.0914	0.3936	0.1530	0.3936	0.0914
Imag	0.0000	-0.0508	0.3458	0.0000	-0.3458	0.0508

Original data as restored by inverse transform

0.3854	0.6772	0.1138	0.6751	0.6362	0.1424
0.5417	0.2983	0.1181	0.7255	0.8638	0.8723
0.9172	0.0644	0.6037	0.6430	0.0428	0.4815
